The Effect of the Electron Temperature on the Electric Polarization Coefficient of Ionospheric Plasma

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Abstract: In this study, the effect of electron temperature on the polarization coefficient of ionospheric plasma has been investigated. The analytical solutions show that if the electron temperature is included to calculation, the ratio of $E_x/E_y$ is complex. If not, it is purely imaginary. It is to say possible that the electron temperature in the ionospheric plasma has been increasing the ratio of $E_x/E_y$ with both local time and the altitude.

Key Words: Electron Temperature, Ionospheric Plasma, Polarization Coefficient

1. Introduction

Numerous authors have studied the propagation of electromagnetic waves and response to the medium of ionospheric plasma [1, 2]. In considering the effect of a continuous distribution of charge on electromagnetic wave propagation, an important quantity is the polarization $P$, which is defined as the induced dipole moment per unit volume of the medium [3]. The propagation of electromagnetic waves depends on the refractive index, i.e. the dielectric properties of medium particularly its conductivity is complex in general [4, 3]. The orientation and behavior of the total electric field is determined by the polarization of the wave [5, 6]. The polarization of a uniform plane wave refers to the behavior of its electric field vector as a function of time at a fixed point in space. The induced dipole moment per unit volume of the medium is very important that it is appears in the dielectric properties of medium [5, 6]. This is a requirement for experimental plasma studies.

The purpose of this study is to investigate the effect on the induced dipole moment per unit volume of the medium of electron temperature in the ionospheric plasma. Also, we are interested in the behavior of dielectric structure of ionospheric plasma when electron temperature is included. Our purpose is to determine the amount of changes in electron temperature the polarization coefficient of wave in the ionospheric plasma. [Ratcliffe had done by neglecting the electron temperature in the ionosphere. But this equation for real cases is absent.]

2. Characteristics of wave polarization in the presence of the electron temperature in an inhomogeneous plasma

The force acting on the electrons in plasma is given by

$$m \ddot{\mathbf{r}} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \mathbf{v} \dot{\mathbf{r}} - \frac{i e^2 k (\mathbf{k} \cdot \mathbf{r})}{\epsilon_0}$$

(1)
Where $\mathbf{E}$, $\mathbf{B}$, $\nu$, $k$, and $c_s$ represent electric field, magnetic field, electron collision frequency, wave vector and the adiabatic speed of sound the electron, respectively\[1, 2, 7\]. It is assumed that the velocity, dipole moment and fields is change such as $e^{i(kr-\omega t)}$, where, $\omega$ is the frequency of wave and $\mathbf{r}$ is the position vector of wave. We assume that the z-axis of the coordinate system with its origin located on the ground is vertical upwards. The x-axis and the y-axis denote geographic east and north in the northern hemisphere respectively. According to the selected coordinate system, the Earth’s magnetic field is assumed to be as in the north hemisphere in Fig (1)[1,2].

![Fig 1. The geometry of wave vector and magnetic field](image)

The magnetic field shown Fig (1) can be as follows.

$$
\mathbf{B} = B_x \mathbf{a}_x + B_y \mathbf{a}_y + B_z \mathbf{a}_z 
$$

where $B_x=B\cos l \sin d$, $B_y=B\cos l \cos d$ and $B_z=-B \sin l$. In this Notation I and d are the magnetic dip and the magnetic declination angles. In this study, magnetic declination angle has been taken as $d=5^\circ$. If there are N electrons per unit volume in plasma, then the dipole moment per unit volume is given by

$$
\mathbf{P} = Ne \mathbf{r} 
$$

If we substitute Eq. (3) into the Eq. (1), the components of the dipole moment per unit volume can be written as follows:

$$
P_x = \frac{\varepsilon_0 X(b_{11} + i b_{12})E_x}{A + iB} + \frac{\varepsilon_0 X(b_{21} + i b_{22})E_y}{A + iB} + \frac{\varepsilon_0 X(b_{31} + i b_{32})E_z}{A + iB} 
$$

$$
P_y = \frac{\varepsilon_0 X(c_{11} + i c_{12})E_x}{A + iB} + \frac{\varepsilon_0 X(c_{21} + i c_{22})E_y}{A + iB} + \frac{\varepsilon_0 X(c_{31} + i c_{32})E_z}{A + iB} 
$$

$$
P_z = \frac{\varepsilon_0 X(d_{11} + i d_{12})E_x}{A + iB} + \frac{\varepsilon_0 X(d_{21} + i d_{22})E_y}{A + iB} + \frac{\varepsilon_0 X(d_{31} + i d_{32})E_z}{A + iB} 
$$

where, $X = \frac{\omega^2_p}{\omega^2}$, and $\omega_p$ and $\omega$ are the plasma frequency and wave frequency respectively. $\varepsilon_0$ is the dielectric constant of free space. (The values of A, B, b, c, d) are provided in appendix. Suppose that, in a plane wave traveling with in z direction, all the components of the wave-fields vary with z like $\exp (-ikz)$, and do not vary in the directions x and y which lie in the wave-front. The ratios of the different components are then the same for values of z so that the wave polarization does not change as travel, and it is a characteristic wave \[18\]. The fourth Maxwell equation can be written as,

$$
\nabla \times \mathbf{H} = \dot{\mathbf{D}} = \varepsilon_0 \dot{\mathbf{E}} + \dot{\mathbf{P}} = \varepsilon \varepsilon_0 \mathbf{E} 
$$

If we solve this equation for the z component shown in Fig (1), the polarization is obtained as,

$$
P_z = -\varepsilon_0 E_z 
$$
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If Eq. (8) is made to Eq(6), $E_z$ becomes as follows

$$E_z = \frac{-(d_{11} + id_{12})XE_x - (d_{21} + id_{22})XE_y}{(A +Xd_{11}) + i(B+Xd_{12})}$$

(9)

If Eq.(9) is substituted into Eq. (4) and Eq. (5), $P_x$ and $P_y$ are obtained as follows:

$$P_x = \chi + \tau \left( \frac{E_y}{E_x} \right)$$

(10)

$$P_y = W \left( \frac{E_x}{E_y} \right) + \alpha$$

(11)

If Eq.(10) is equalized to Eq.(11), The expression for the ratio of $\frac{E_x}{E_y}$ can be expressed in the following equation:

$$W \left( \frac{E_x}{E_y} \right)^2 - \left( \chi - \tau \right) \left( \frac{E_x}{E_y} \right) - \tau = 0$$

(12)

If Eq.(12) is solved to find the roots of the equation,

$$\frac{E_x}{E_y}_{1,2} = \frac{(\chi - \alpha) \pm \sqrt{(\alpha - \chi)^2 + 4W\tau}}{2W}$$

(13)

When these constants ($\tau$, $\chi$, $\alpha$ and $W$) are substituted into Eq.(13), we obtain.

$$\frac{E_x}{E_y}_{1,2} = \frac{(\chi_{11} - \alpha_{11}) + i(\chi_{12} - \alpha_{12}) \pm [\lambda + i\gamma]^\frac{1}{2}}{2(W_{11} + iW_{12})}$$

(14)

where

$$\lambda = \alpha_{11}^2 - \alpha_{12}^2 + \chi_{11}^2 - \chi_{12}^2$$

$$+ 2(\alpha_{11}\chi_{11} - \alpha_{12}\chi_{12}) + 4(W_{11}\tau_{11} - W_{12}\tau_{12})$$

$$\gamma = 2(\alpha_{11}\alpha_{12} + \chi_{11}\chi_{12})$$

$$- 2(\alpha_{11}\chi_{12} + \alpha_{12}\chi_{11}) + 4(W_{11}\tau_{12} + W_{12}\tau_{11})$$

The expression in the square root of Eq. (14) can be expanded as follows:

$$\frac{[\lambda + i\gamma]^\frac{1}{2}}{2} = K + iL$$

(15)

and the following can be deduce

$$K_{1,2} = \frac{1}{2} \left( \lambda \pm \sqrt{\lambda^2 + \gamma^2} \right)$$

(16)

$$L_{1,2} = \frac{1}{2} \left( -\lambda \pm \sqrt{\lambda^2 + \gamma^2} \right)$$

(17)

If Eq.(15) and Eq.(16) are substituted into Eq. (14), the ratio $\frac{E_x}{E_y}_{1,2}$ can be obtained as follows:

$$\frac{E_x}{E_y}_{1,2} = \frac{W_{11}[(\chi_{11} - \alpha_{11}) + W_{12}[(\chi_{12} - \alpha_{12}) \pm [W_{11}K_{1,2} + W_{12}L_{1,2}]]}{2(W_{11}^2 + W_{12}^2)}$$

$$+ i\frac{W_{11}[(\chi_{12} - \alpha_{12}) - W_{12}[(\chi_{11} - \alpha_{11}) \pm [W_{11}L_{1,2} - W_{12}K_{1,2}]]}{2(W_{11}^2 + W_{12}^2)}$$

$$W = \frac{W_{11} + iW_{12}}{M + iN}$$

(18)

For the abbreviations in the Eq.(18), see the

$$\tau = \frac{\tau_{11} + i\tau_{12}}{M + iN}$$

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Appendix.

3. Numerical solutions and analysis

The calculations of the equations have been carried out for geographic coordinates (39° N-40° E, I=55.7° and d=5°). The used plasma parameters have been calculated by using IRI-90 for June at 1200 LT. In the calculations, the sunspot number R is taken as 10. The wave frequencies are selected around the plasma frequency ($\omega \approx 3-6 \times 10^6$ rad/sn). The analytical solutions show that if the electron temperature is included to calculation, the ratio of $E_x/E_y$ is complex. If not, it is pure imaginary. That is, when the electron temperature is considered, the wave has elliptical polarization. Otherwise, the wave has circular polarization.

4: Conclusions

We can say that the $E_x/E_y$ increases, if electron density increases in the ionosphere plasma. Values that the ratio of $E_x/E_y$ has maximum is peak height (hmF\text{\~}F \approx 255\text{km}) in the ionosphere plasma (Fig 2.). This case is the same for local time. That is, the change of difference [$=\Delta(E_x/E_y)(T_e\neq0)-(T_e=0)$] with local time is minimum as the electron density has minimum(Fig 3.).

According to the obtained results, the electron temperature raises $E_x/E_y$. This increase has been affecting the behavior of both electromagnetic wave and medium such as the dielectric structure of medium, the direction of electromagnetic wave, because the behavior of electromagnetic waves in ionospheric plasma directly depends on the refractive index of medium. Moreover, the polarization coefficient indirectly determines the refractive index of medium such as ionosphere. This case is crucial on HF propagation in ionosphere.

![Fig.2. The change of difference=$\Delta(E_x/E_y)(T_e\neq0)-(T_e=0)$] with the altitude.](image-url)
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Fig.3. The change of difference $\Delta(E_x/E_y)\left(\left(T_e\neq 0\right) - \left(T_e = 0\right)\right)$ with local time.

5. References

Appendix

\[ A = 1 - 3Z^2 - a(1 - Z^2) - Y^2(1 - a\sin^2\theta) \quad B = Z(3 - 2a - Z^2 - Y^2) \]

\[ b_{11} = 1 - Z^2 - a - Y^2\cos^2\sin^2d \quad b_{12} = 2Z - aZ \]

\[ b_{21} = YZ\sin I - Y^2\cos^2\cos\sin\sind \quad b_{22} = (a - 1)Y\sin I \]

\[ b_{31} = YZ\cos I\cos d + Y^2\sin I\sin\sind \quad b_{32} = Y\cos I\cos d \]

\[ c_{11} = -YZ\sin I - Y^2\cos^2\cos\sin\sind \quad c_{12} = (1 - a)Y\sin I \]

\[ c_{21} = 1 - a - Z^2 - Y^2\cos^2\cos^2d \quad c_{22} = 2Z - aZ \]

\[ c_{31} = Y^2\cos I\sin I\cos d - YZ\cos I\sin d \quad c_{32} = Y\cos I\sin d \]

\[ d_{11} = -YZ\cos I\cos d + Y^2\cos I\sin I\sin d \quad d_{12} = Y\cos I\cos d \]

\[ d_{21} = Y^2\cos I\sin I\cos d + YZ\cos I\sin d \quad d_{22} = -Y\cos I\sin d \]

\[ d_{31} = 1 - Z^2 - Y^2\sin^2 I \quad d_{32} = 2Z \]

\[ \chi = \left( \frac{(b_{11} + ib_{12})}{A + iB} \right) \epsilon_0 X - \left( \frac{(b_{31} + ib_{32})}{A + iB} \right) \epsilon_0 X^2 \left( \frac{(d_{11} + id_{12})}{A + iB} \right) + \left( \frac{(d_{31} + id_{32})}{A + iB} \right) \epsilon_0 \left( \frac{X^2}{(A + iB)((A + Xd_{31}) + i(B + Xd_{32}))} \right) \]

\[ \tau = \left( \frac{(b_{21} + ib_{22})}{A + iB} \right) \epsilon_0 X - \left( \frac{(b_{31} + ib_{32})}{A + iB} \right) \epsilon_0 X^2 \left( \frac{(d_{21} + id_{22})}{A + iB} \right) + \left( \frac{(d_{31} + id_{32})}{A + iB} \right) \epsilon_0 \left( \frac{X^2}{(A + iB)((A + Xd_{31}) + i(B + Xd_{32}))} \right) \]

\[ \zeta = \left( \frac{(c_{11} + ic_{12})}{A + iB} \right) \epsilon_0 X - \left( \frac{(c_{31} + ic_{32})}{A + iB} \right) \epsilon_0 X^2 \left( \frac{(d_{11} + id_{12})}{A + iB} \right) + \left( \frac{(d_{31} + id_{32})}{A + iB} \right) \epsilon_0 \left( \frac{X^2}{(A + iB)((A + Xd_{31}) + i(B + Xd_{32}))} \right) \]

\[ \zeta = \left( \frac{(c_{21} + ic_{22})}{A + iB} \right) \epsilon_0 X - \left( \frac{(c_{31} + ic_{32})}{A + iB} \right) \epsilon_0 X^2 \left( \frac{(d_{21} + id_{22})}{A + iB} \right) + \left( \frac{(d_{31} + id_{32})}{A + iB} \right) \epsilon_0 \left( \frac{X^2}{(A + iB)((A + Xd_{31}) + i(B + Xd_{32}))} \right) \]

\[ \chi_{11} = \epsilon_0 X(b_{11}(A + Xd_{31}) - b_{11}(B + Xd_{32})) - \epsilon_0 X^2(b_{31}d_{11} - b_{32}d_{12}) \]

\[ \chi_{12} = \epsilon_0 X[b_{11}(A + Xd_{31}) - b_{12}(A + Xd_{32})] - \epsilon_0 X^2(b_{31}d_{12} + b_{32}d_{11}) \]

\[ \tau_{11} = \epsilon_0 X[b_{21}(A + Xd_{31}) - b_{22}(A + Xd_{32})] - \epsilon_0 X^2(b_{31}d_{21} - b_{32}d_{22}) \]

\[ \tau_{12} = \epsilon_0 X[b_{21}(B + Xd_{32}) + b_{22}(A + Xd_{31})] - \epsilon_0 X^2(b_{32}d_{21} + b_{31}d_{22}) \]

\[ W_{11} = \epsilon_0 X[c_{11}(A + Xd_{31}) - c_{12}(B + Xd_{32})] - \epsilon_0 X^2(c_{31}d_{11} - c_{32}d_{12}) \]

\[ W_{12} = \epsilon_0 X[c_{11}(B + Xd_{32}) + c_{12}(A + Xd_{31})] - \epsilon_0 X^2(c_{31}d_{12} + c_{32}d_{11}) \]

\[ M = A(A + Xd_{31}) - B(B + Xd_{32}) \]

\[ N = A(B + Xd_{32}) + B(A + Xd_{31}) \]

\[ a = \frac{k^2c^2_s}{\omega^2} \quad \text{and} \quad c^2_s = \frac{\gamma k_b T_e}{m_e} \]