International Journal of Science & Technology
Volume 1, No 1, 43-50, 2006

Large Deflection Static Analysis of Rectangular Plates On Two Parameter Elastic Foundations

Omer CIVALEK[1] and Altug YAVAS[2]
[1]Akdeniz University, Faculty of Engineering, Civil Engineering Dept., Antalya, TURKEY
[2]Balikesir University, Faculty of Engineering, Civil Engineering Dept., Balikesir, TURKEY
civalek@yahoo.com

Abstract: The main purpose of this paper is the application of the discrete singular convolution (DSC) method to solid mechanics. Geometrically nonlinear static analysis of thin rectangular plates on Winkler-Pasternak elastic foundation has been studied. The nonlinear partial differential equations obtained from Von Karman's large deflection plate theory have been solved by using the DSC method. The effects of Winkler and Pasternak foundation parameters on the displacements have been investigated. The accuracy of the proposed DSC method is demonstrated by the numerical examples.

Keywords: Plates, nonlinear analysis, discrete singular convolution,deflections.

İki Parametreli Elastik Zemine Oturan Büyük Deformasyonlu Dikdörtgen Plakların Statik Hesabı

Özet: Bu çalışmanın esas amacı discrete singular konvolution (DSC) metodunun katı mekaniğine uygulanmasıdır. Winkler-Pasternak elastik zemine oturan ince dikdörtgen plakların geometrik bakımdan lineer olmayan analizi sunulmuştur. Von Karmanın büyük deformasyonlu plak teorisine bağlı olarak çıkartılan diferansiyel denklem DSC metodu kullanılarak çözülmüştür. Winkler ve Pasternak zemin parametrelerinin deplasmanlar üzerine etkisi incelenmiştir. Sayısal örnekler ile metodun yaklaşımı gösterilmiştir.

Anahtar Kelimeler: Plak, lineer olmayan analiz, ayrık tekil konvolisyon, deplasman.

1. Introduction

Nonlinear static and dynamic analysis of plates of various shapes has been carried out by various researchers [1-5]. More detailed information can be found in a recent review paper by Sathyamorth [6]. Nath et al. [7,8] presented the finite differences methods for spatial discretization and Houbolt's time marching discretization to study the dynamic analysis of rectangular plates resting on elastic foundation. Dumir [9] and Dumir and Bhaskar [10] have investigated nonlinear static and dynamic analysis of rectangular plates on elastic foundation employing the orthogonal point collocation method. Civalek [11,12,15,16] and Civalek and Ülker [13,14] applied harmonic differential quadrature (HDQ) method to the solutions of plates with or without on elastic foundation.

2. Discrete singular convolution (DSC)

The discrete singular convolutions (DSC) algorithm was originally introduced by Wei [17] as a simple and highly efficient numerical technique. In this paper, details of DC method are not given; interested readers may refer to the works of Wei [18,19] and Wei et. al.[20] who originated the method. Wei and his co-workers first applied the DSC algorithm to solve solid and fluid mechanics problem [21-23]. These studies indicate that the DSC algorithm works very well for the vibration analysis of plates, especially for high-frequency analysis of rectangular plates. Consider a distribution, T and $\eta(t)$ as an element of the space of the test function. A singular convolution can be defined by

$$F(t) = (T * \eta)(t) = \int_{-\infty}^{\infty} T(t-x)\eta(x)dx \qquad (1)$$

where $T(t-x)$ is a singular kernel. The DSC algorithm can be realized by using many approximation kernels. However, it was shown [17-20] that for many problems, the use regularized Shannon kernel (RSK) is very efficient. The RSK is given by

$$\delta_{\Delta,\sigma}(x-x_k)=\frac{\sin[(\pi/\Delta)(x-x_k)]}{(\pi/\Delta)(x-x_k)}\exp\left[-\frac{(x-x_k)^2}{2\sigma^2}\right] \quad (2)$$

where $\Delta=\pi/(N-1)$ is the grid spacing and N is the number of grid points. The parameter σ determines the width of the Gaussian envelop and often varies in association with grid spacing, i.e., $\sigma = r$h. Here r is a parameter chosen in computation. It is also known that the truncation error is very small due to the use of the Gaussian regularizer, the above formulation given by Eq. (2) is practically and has an essentially compact support for numerical interpolation. With a sufficiently smooth approximation, it is more effective to consider a discrete singular convolution

$$F_\alpha(t)=\sum_k T_\alpha(t-x_k)f(x_k) \quad (3)$$

where F_α (t) is an approximation to $F(t)$ and $\{x_k\}$is an appropriate set of discrete points on which the DSC is well defined in Eq. (1). Note that, the original test function $\eta(x)$ has been replaced by f(x). This new discrete expression is suitable for computer realization. The mathematical property or requirement of f(x) is determined by the approximate kernel T_α . In the DSC method, function f (x) and its derivatives with respect to x coordinate at a grid point x_i are approximated by a linear sum of discrete values f (x_k) in a narrow bandwidth [x-x_M, x+x_M]. This can be expressed as

$$f^{(n)}(x)\approx\sum_{k=-M}^{M}\delta^{(n)}_{\Delta,\sigma}(x_i-x_k)f(x_k)\,; \quad (n=0,1,2,...,) \quad (4)$$

where superscript n denotes the nth-order derivative with respect to x. The x_k is a set of discrete sampling points centered around the point x, σ is a regularization parameter, Δ is the grid spacing, and 2M+1 is the computational bandwidth, which is usually smaller than the size of the computational domain. The higher order derivative terms $\delta^{(n)}_{\Delta,\sigma}(x-x_k)$ in Eq.(2) are given as below;

$$\delta^{(n)}_{\Delta,\sigma}(x-x_k)=\left(\frac{d}{dx}\right)^n[\delta_{\Delta,\sigma}(x-x_k)] \quad (5)$$

where, the differentiation can be carried out analytically.

3. Governing equations

The geometry of a typical rectangular plate resting on Winkler elastic foundation is shown in Fig. 1. The foundation is modeled in terms of Winkler parameter k_f. Including the normal inertia and neglecting damping of the foundation, the distributed reaction from the elastic foundation on the shell at any instant of time t is given by

$$k_f w+\rho_f h_f w_{tt}\,, \quad (6)$$

Considering the plate-foundation interaction and neglecting the in-plane and rotary inertia, the governing differential equations of motion in terms of non-dimensional displacements components U, V, and W for geometrically nonlinear dynamic analysis of thin rectangular plates resting on Winkler elastic foundation can be expressed as [8]:

$$U_{,XX}+\frac{\beta^2}{2}(1-\upsilon)U_{,YY}+\frac{\beta}{2}(1+\upsilon)V_{,XY}+\left[W_{,XX}+\frac{\beta^2}{2}(1-\upsilon)W_{,YY}\right]W_{,X}+\frac{\beta^2}{2}(1+\upsilon)W_{,YY}W_{,XY}=0\,, \quad (7)$$

$$\beta^2V_{,YY}+\frac{1}{2}(1-\upsilon)V_{,XX}+\beta\frac{1}{2}(1+\upsilon)U_{,XY}+\beta\left[\beta^2W_{,YY}+\frac{1}{2}(1-\upsilon)W_{,XX}\right]W_{,Y}+\beta\frac{1}{2}(1+\upsilon)W_{,X}W_{,XY}=0\,, \quad (8)$$

$$W_{,XXXX}+2\beta^2W_{,XXYY}+\beta^4W_{,YYYY}-12\left[U_{,X}+\beta\upsilon V_{,Y}+\frac{1}{2}(W_{,X})^2+\frac{1}{2}\beta^2\upsilon(W_{,Y})^2\right](W_{,XX})$$

$$-\left[\beta V_{,Y}+\upsilon U_{,X}+\frac{1}{2}\upsilon(W_{,X})^2+\frac{1}{2}\beta^2(W_{,Y})^2\right]$$

$$\times\left[-12(\beta^2 W_{,YY})\right]$$

$$-12\left[\beta U_{,Y}+V_{,X}+\beta W_{,X}W_{,Y}\right](1-\upsilon)\beta W_{,XY}$$

$$+KW=12(1-\upsilon^2)P \qquad (9)$$

The non-dimensional quantities in the above equations are defined as

$$W=w/h,\ X=x/a,\ Y=y/b,\ \beta=a/b,$$
$$U=ua/h^2, V=va/h^2,$$
$$D=Eh^3/12(1-\upsilon^2),\ K=k_f a^4/D,$$
$$P=qa^4/Eh^4. \qquad (10)$$

where u, v and w are displacement components in the x, y, and z directions, respectively, h and h_f are the thickness of the plate and foundation, E is Young's modulus, ν is Poisson's ratio, ρ_f and ρ are the mass density of the foundation and the material, K is the stiffness parameters of Winkler foundation, D is the flexural rigidity, a and b are the sides of plate along x and y directions, t is the time.

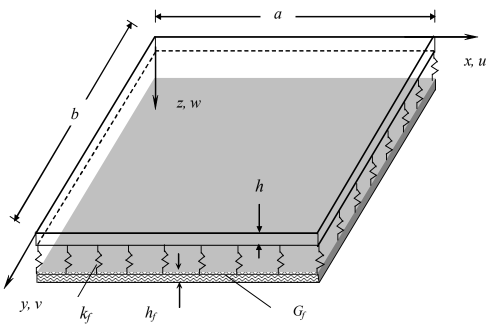

Fig. 1. Geometry and dimensions of rectangular plates on elastic foundation.

In the present study the following two types of boundary conditions are considered. For all simply supported four edges and immovably constrained against in-plane translation (SSSS):

$$U=V=W=0 \qquad (11)$$

$$\left(\frac{\partial^2 W}{\partial X^2}+\upsilon\beta^2\frac{\partial^2 W}{\partial Y^2}\right)=0 \qquad (12)$$

$$U=V=W=0 \qquad (13)$$

$$\left(\beta^2\frac{\partial^2 W}{\partial Y^2}+\upsilon\frac{\partial^2 W}{\partial X^2}\right)=0 \qquad (14)$$

at $X=0$ and $X=1$ and at $Y=0$ and $Y=1$. For all clamped four edges and immovably constrained against in-plane translation (CCCC):

$$U=V=W=0 \qquad (15)$$

$$\left(\frac{\partial W}{\partial X}\right)=0 \qquad (16)$$

$$U=V=W=0 \qquad (17)$$

$$\left(\frac{\partial W}{\partial X}\right)=0 \qquad (18)$$

at $X=0$ and $X=1$ and at $Y=0$ and $Y=1$. In this study, a uniform grid in both the x- and y-directions is employed (i.e. $\Delta_x=\Delta_y=\Delta$). We denote a grid point (x_i, y_j) by $x_i = i\Delta_x$ and $y_i = i\Delta_y$ and the differences of two grid points by $x_i - x_{i+k} = -k\Delta_x$ and $y_i - y_{i+k} = -k\Delta_y$. The value of U and V at a grid point (x_i, y_j) are denoted by $U_{i,j}$ and $V_{i,j}$, respectively. After spatial discretization, DSC form of the governing equations and boundary conditions are given as

$$\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta)U_{i+k,j}+\beta^2 d_1\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta)U_{i,j+k}$$

$$+\beta d_2\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)V_{i+k,j}\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)V_{i,j+k}$$

$$+\left(\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta)W_{i+k,j}\right)\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)W_{i+k,j}$$

$$+\left(d_1\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta)W_{i,j+k}\right)\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)W_{i+k,j}$$

$$+\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)W_{i+k,j}\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)W_{i,j+k}$$

$$\times\left(\beta^2 d_2\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta)W_{i,j+k}\right)=0 \quad (19)$$

$$\beta^2\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta)V_{i,j+k}+d_1\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta)V_{i+k,j}$$

$$+\beta d_2\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)U_{i+k,j}\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)U_{i,j+k}$$

$$+\left(\beta^2\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta)W_{i,j+k}\right)\beta\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)W_{i,j+k}$$

$$+\left(d_1\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta)W_{i+k,j}\right)\beta\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)W_{i,j+k}$$

$$+\left(\beta d_2\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)W_{i+k,j}\right)$$

$$\times\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)W_{i+k,j}$$

$$\times\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)W_{i,j+k}=0 \quad (20)$$

$$\sum_{k=-M}^{M}\delta^{(4)}_{\Delta,\sigma}(k\Delta)W_{i+k,j}+\beta^4\sum_{k=-M}^{M}\delta^{(4)}_{\Delta,\sigma}(k\Delta)W_{i,j+k}$$

$$+2\beta^2\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta)W_{i+k,j}\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta)W_{i,j+k}$$

$$-12\left\{\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)U_{i+k,j}+\right.$$

$$\beta\upsilon\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)V_{i,j+k}\frac{1}{2}\left(\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)W_{i+k,j}\right)^2$$

$$\left.+\beta^2\frac{\upsilon}{2}\left(\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)W_{i,j+k}\right)^2\right\}\times\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta)W_{i+k,j}$$

$$-12\left\{\beta\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)V_{i,j+k}+\right.$$

$$+\upsilon\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)U_{i+k,j}$$

$$-\frac{\upsilon}{2}\left(\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)W_{i+k,j}\right)^2$$

$$\left.+\beta^2\frac{\upsilon}{2}\left(\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)W_{i,j+k}\right)^2\right\}$$

$$\times\beta^2\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta)W_{i,j+k}$$

$$-12\left\{\beta\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)U_{i,j+k}\right.$$

$$+\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)V_{i+k,j}$$

$$\left.++\beta\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)W_{i+k,j}\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)W_{i,j+k}\right\}$$

$$\times\left(2d_1\beta\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)W_{i+k,j}\right)$$

$$\times\left(\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)W_{i,j+k}\right)+KW_{i,j}$$

$$= 12P(1-\upsilon^2) \tag{21}$$

where $\delta^{(1)}_{\Delta,\sigma}$, $\delta^{(2)}_{\Delta,\sigma}$, $\delta^{(3)}_{\Delta,\sigma}$ and $\delta^{(4)}_{\Delta,\sigma}$ are coefficients of the regularized Shannon's delta kernel for spatial discretization using DSC. The set of non-linear algebraic equations (19-21) can be solved for $\{U\}$, $\{V\}$ and $\{W\}$ using non-linear algorithms such as Newton-Raphson method [4, 15].

4. Numerical results

The title problem is analysed and some of DSC-HDQ results are compared with results in the open literature [7-10] to show the applicability and efficiency of DSC-HDQ coupled methodology. To check whether the purposed formulation and programming are correct, a clamped immovable plates without an elastic foundation is analysed first. The load-central displacement curve of clamped immovable rectangular (b/a=0.5) and square plate(b/a=1) under uniform distributed load is compared in Fig.2 with the results of Dumir and Bhaskar[10]. The DSC results based on regularized Shannon's delta (RSD) kernel are generally in agreement with the results of Rererence [10]. The performance of the DSC is excellent.

In the following examples, unless specified, the RSD kernel based DSC is used for spatial discretization and the HDQ method is used for time discretization. Furthermore, the computational bandwidth is chosen as 16. The regularization parameter is set to $\sigma = 2.46\Delta$.

The static response of clamped and simply supported immovable square plates on Winkler-Pasternak foundations is depicted in Figs 3 and 4 for various values of the foundation parameters, *K* and *G*. The obtained results agree excellently with those of Dumir and Bhaskar[10] solution. It may be concluded that increasing the applied load will always result in increased displacements. It is shown that the S-S boundary condition presents larger displacements values than the C-C boundary condition. Thus, it can be concluded that the results are also dependent on the boundary conditons of the plates. The effect of *K* on the response of clamped supported plates resting on Winkler elastic foundation is shown in Figure 5. Figure 5 show that deflections will decrease with increase in foundation parameter. It is also observed that the influence of Winkler parameter *K*, on the displacements is significant. Variation of maximum deflection of clamped rectangular plates with different values of Winkler modulus *K* as depicted in Fig. 6. The influence of the foundation parameters, namely *K*, has been studied for statical analysis. It is shown that the response decrease as the *K* increases.

5. Conclusions

The present paper focusses on the application of DSC method. For this purpose geometrically non-linear static analysis is studied for thin rectangular plates on elastic foundation. To the authors' knowledge, it is the first time the DSC method has been successfully applied to rectangular plates resting on two parameter elastic foundation problem for the geometrically nonlinear static analysis. It is appeared that the parameter *K* and *G* of the Winkler and Pastenak foundation has been found to have a significant influence on the displacements of the plates. In fact, similar results were previously found. Consequently, by comparing the computed results with those avaliable in published works, the present analysis by the DSC method is examined and a very good agreement is observed.

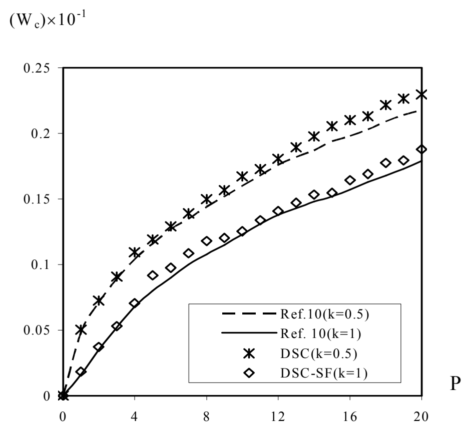

Fig. 2. Load- Central deflection response of clamped plate (ν=0.3).

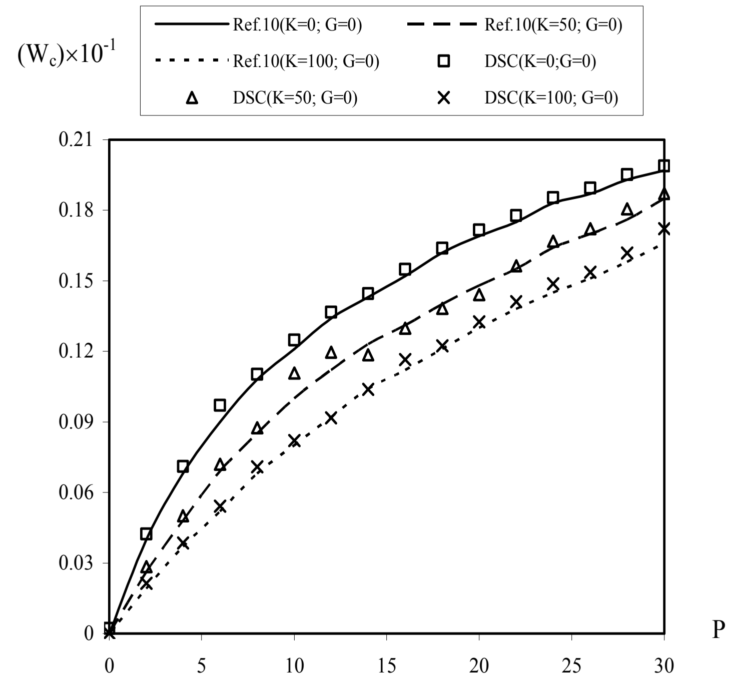

Fig. 3. Central deflection versus load with different foundation parameters (CCCC Plate)

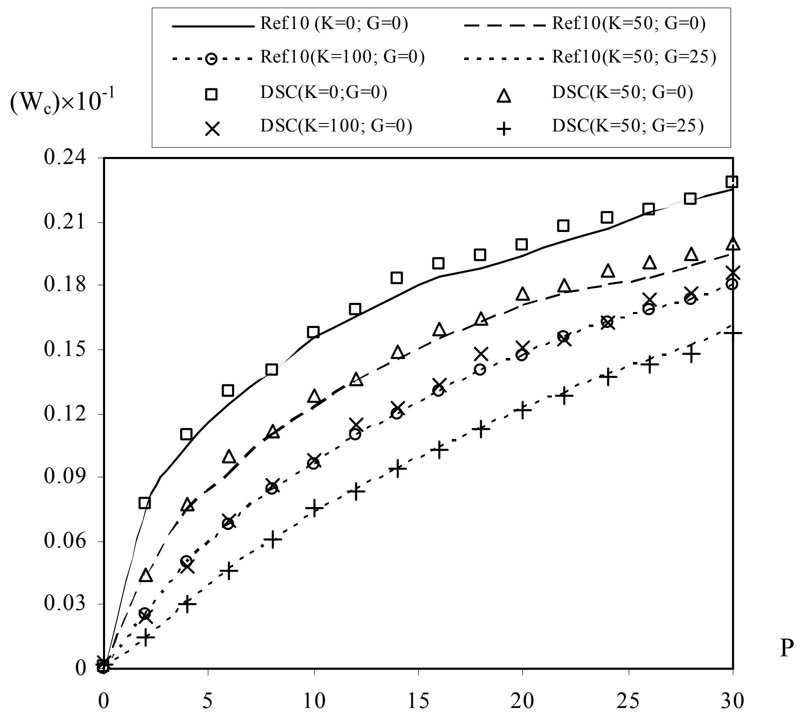

Fig. 4. Central deflection versus uniform load with different foundation parameters (SSSS Plate).

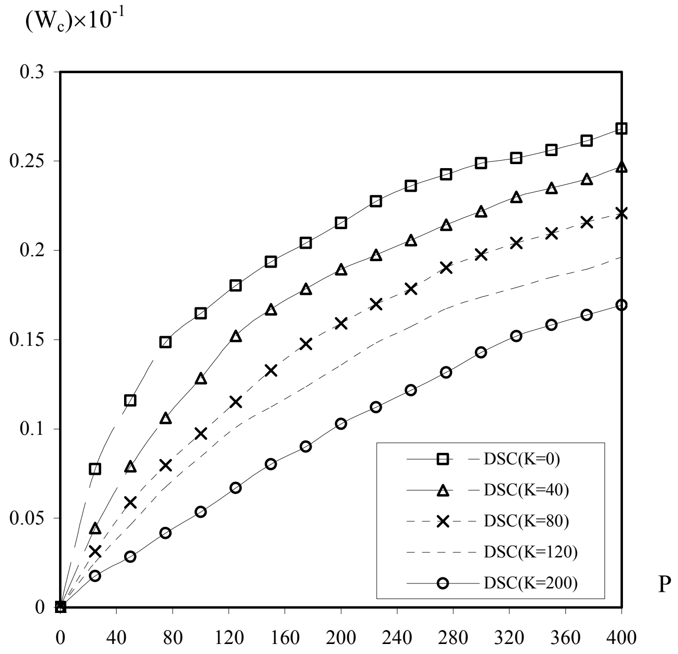

Fig. 5. Variation of central deflection with different load and foundation parameters for CCCC rectangular plates (b/a=2; ν=0.3).

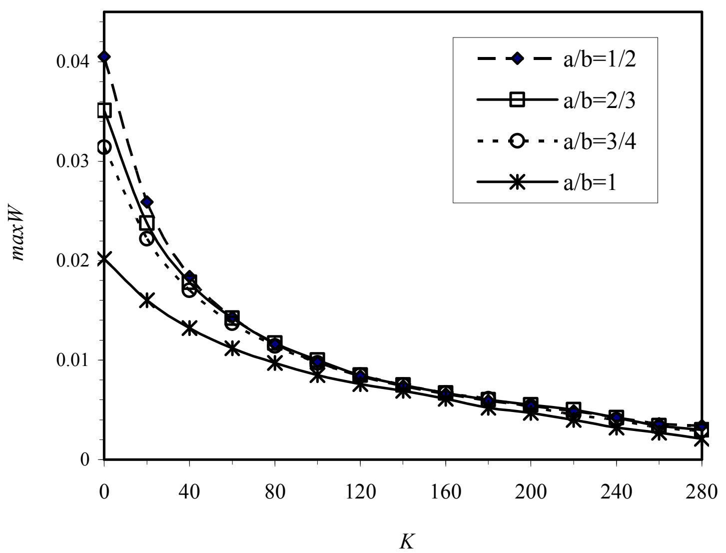

Fig. 6. Variation of maximum deflection of clamped plates with Winkler modulus

Acknowledgements

The financial support of the Scientific Research Projects Unit of Akdeniz University is gratefully acknowledged.

6. References

1. Leissa, A.W. (1973).Vibration of plates. NASA, SP-160.
2. Chia, C.Y. (1980). Nonlinear analysis of plates. Mc-Graw Book Co., New York, N.Y.
3. Soedel, W. (1996). Vibrations of shells and plates. Second Edition, Revised and Expanded, Marcal Dekker, Inc., New York.
4. Civalek, Ö.(1998). Finite Element analysis of plates and shells. Elazığ: Fırat University (in Turkish).
5. Leissa, A.W. (1973). The Free Vibration of Rectangular Plates. *J. Sound and Vibration*, **31**,257-293, 1973.
6. Sathyamorth, M. (1987). Nonlinear vibration analysis of plates: A review and survey of current developments. *Appl. Mech. Rev*., **40**,1533-1561.
7. Nath, Y. and Kumar, A.(1995). Chebyshev series solution to nonlinear boundary value problems in rectangular domain. *Comput. Method. Appl. Mech. and Eng*. **125**,41-52.
8. Nath, Y. Varma, K.K. and Mahrenholtz, D. (1986) Nonlinear Dynamic response of rectangular plates on linear elastic foundation, *Computers and Structures*, **24**(3),391-399.
9. Dumir, P.C. (1988). Nonlinear dynamic response of isotropic thin rectangular plates on elastic foundations, *Acta Mechanica*, **71**,233-244.
10. Dumir P.C. and Bhaskar, A. (1988). Nonlinear static analysis of rectangular plates on elastic foundations by the orthogonal point collocation method, *Comput. Method. Applied Mech. and Eng*. **67**, 111-124.
11. Civalek, Ö.(2004). Application of differential quadrature (DQ) and harmonic differential quadrature (HDQ) for buckling analysis of thin isotropic plates and elastic columns, *Engineering Structures, An International Journal*, **26**(2), 171-186.
12. Civalek, Ö.(2005). Geometrically nonlinear dynamic analysis of doubly curved isotropic shells resting on elastic foundation by a combination of HDQ- FD methods, *International Journal of Pressure Vessels and Piping,* 82(6) 470-479.
13. Civalek, Ö., Ülker,M.(2004). Harmonic differential quadrature (HDQ) for axisymmetric bending analysis of thin isotropic circular plates, *International Journal of Structural Engineering and Mechanics*, Vol. **17**(1), 1-14.
14. Civalek, Ö., Ülker,M.(2004). Free vibration analysis of elastic beams using harmonic differential quadrature (HDQ), *Mathematical and Comp. Appl*. **9**(2) 257-264.
15. Ö. Civalek,(2004). Polinomal Diferansiyel Quadrature (PDQ) Metodu ile Elastik Zemine Oturan Plak ve Kabukların Geometrik Bakımdan Lineer Olmayan Statik ve Dinamik Analizi,Doktora Tezi, Fırat Üniversitesi Fen Bilimleri Enstitüsü.
16. Ö. Civalek,(2003). Çok Serbestlik Dereceli Sistemlerin Harmonik Diferansiyel Quadrature (HDQ) Metodu ile Lineer ve Lineer Olmayan Dinamik Analizi, Dokuz Eylül Üniversitesi Fen Bilimleri Enstitüsü.
17. Wei, G.W. (2001). A new algorithm for solving some mechanical problems, *Comput. Methods Appl. Mech. Engng*, **190**,2017-2030.
18. Wei, G.W. (2001). Vibration analysis by discrete singular convolution, *Journal of Sound and Vibration*, **244**, 535-553.
19. Wei, G.W. (2001). Discrete singular convolution for beam analysis, *Engineering Structures*, **23**, 1045-1053.
20. Wei, G.W. Zhou, Y.C. and Xiang, Y.(2002). Discrete singular convolution and its application to the analysis of plates with internal supports. Part 1: Theory and algorithm. *Int J Numer Methods Eng.* **55**,913-946.
21. Wei, G.W. (1999). Discrete singular convolution for the solution of the Fokker –Planck equations. *J Chem Phys*, **110**,8930-8942.
22. G Wei, G.W. Zhou, Y.C. and Xiang, Y.(2002). A novel approach for the analysis of high-frequency vibrations, *Journal of Sound and Vibration*, **257**(2), 207-246.