Stability Analysis of Earth Retaining Walls

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Abstract—Stabilization of the slopes by the technique of reinforced earth (Terre Armée) is a very concurrent technique (economical and reliable). Based on the results of theoretical and experimental studies, we propose in this paper to check the overall internal stability of reinforced earth retaining walls by three mechanical models, using the analytical method of the limit equilibrium (failure). The main objective of this paper is to compare these failure mechanical models with the failure models obtained by numerical analysis (code FLAC2d), in order to validate the most realistic and more unfavourable failure models. Parametric and comparative studies carried out have allowed us to bring a very useful knowledge concerning the study of internal stability of the reinforced earth retaining walls and to propose a theoretical mechanical model of calculation proven by numerical simulation and confirmed by tests.

Keywords—Retaining wall, Earth wall, Mechanical behaviour, internal stability, limit equilibrium.

I. INTRODUCTION

The instabilities of slopes constitute always the main risks on human lives and loss of goods. The retaining walls are conceived to retain unstable slopes. There is a large variety of these structures according to the method of their construction and their mechanical behaviour. In 1960, Henri Vidal invents the fundamental mechanism of the reinforced earth and introduced for the first time the “Terre Armée” as an alternative type of the retaining walls ((1, 2)). The reinforcements (metallic strips, geosynthetics, tires, …) are placed inside the soil mass. The main advantages of the reinforced earth, which explains its significant development in France and worldwide, are its economy, its integration in the ground structures (in the case of road embankments in particular) and especially its great deformability which enables it to adapt without risk to important motions. The reinforcements resist tensile, shearing and/or flexural forces, according to their type, by friction ground-reinforcements. The reinforced earth walls behave mechanically like a weight-wall, using their weight to withstand the earth pressures.

On the basis of full-scale and small-scale tests, we distinguish the following internal failure modes of the reinforced earth retaining walls ((1, 3)):

- failure by break of the reinforcements,
- failure by loss of adherence (pullout),
- failure of the facing,
- overall failure (strip-ground-facing).

The main aim of this paper is to analyse the latter failure mode (overall failure of the structure). To avoid the overall internal failure (ground-strip-facing), we have to determine the maximum tensile forces developed in the reinforcements and the geometry of the critical slip surface. In this paper, we have studied this type of stability with the traditional method of soil mechanics (limit state equilibrium) [4] and then confronted it with the results of numerical simulation. The general method of checking of the internal stability is summarized in the following steps:

- determine the critical slip surface (mechanical model),
- determine the maximum tensile force on each level of reinforcement,
- determine the maximum tensile force by defect of adherence and by break at each level of reinforcement,
- evaluate the safety factor.

II. LIMIT EQUILIBRIUM METHODS

For all the techniques of the reinforced earth walls, tensile forces in the reinforcements are not maximum at the face but inside the reinforced soil mass. The locus of the points of maximum tension $T_{max}$ separates the soil mass in two zones (Fig. 1): an active zone located behind the facing where shear stresses at the interface soil-reinforcement are directed towards the outside and a resistant zone where shear stresses are directed towards the interior and are opposed to the side displacement of the active zone ((3, 5-7)).

The method of local equilibrium, which was developed for the first time for the reinforced earth “Terre Armée”, consists in studying the equilibrium of a section of ground and facing.
around a horizontal element of reinforcement (Fig. 1). It is classically supposed that the shear stresses on the upper and lower faces equilibrate as well as the horizontal sharp efforts in the facing. Therefore, the shearing is null at the point of maximum tension $T_{\text{max}}$ ($T$ is maximum and its derivative, proportional to $\tau$, is null) and both the horizontal and vertical directions are principal directions for the stresses. The back face of the section is then taken vertical at the point of maximum tension $T_{\text{max}}$, which makes it possible to simply write the horizontal balance of the section in the following form

$$T_{\text{max}} = S, S_h K \sigma (z)$$

(1)

where $S$, and $S_h$ are vertical and horizontal spacing of the reinforcements (Fig. 1); $\sigma(z)$ is vertical stress at depth $Z$ and at the point of maximum tension whose distribution along a horizontal reinforcement is supposed to be non uniform (Fig. 1); $K$ is coefficient relating the horizontal stress to the vertical stress, it can be determined as an average coefficient of earth pressure along the line of $T_{\text{max}}$ and by the distribution of vertical stress $\sigma(z)$ in relation to the depth.

The method of total equilibrium consists of considering plans of potential failure resulting from any point of the facing corresponding to failure wedges (Fig. 2) [3].

### III. MECHANICAL FAILURE MODELS

The stability conditions of a reinforced earth wall are strongly related to the geometry, the properties of mechanical resistance of the ground, the reinforcement and the ground- strip interaction. The principle of the detection of risks of failure is summarized in two steps:

- the objective of the first step is to identify the geometrical configurations favourable to failure for the various known failure mechanisms (plane failure, circular failure and mixed failure),
- the second step implies the calculation of the safety factors associated to each failure mechanism identified at the preceded step.

We assume that the most critical slip surface by a reinforced earth wall coincides with the line of maximum tensile forces (i.e. the locus of maximum tensile force $T_{\text{max}}$ in each layer of reinforcements). In this paper, we have studied three mechanical failure models of reinforced earth retaining walls which are illustrated in Fig. 2.

We cut out the soil mass in a number of elementary volumes (slides), for each of these the line of slip is a straight one, i.e. we discreate the failure surface in segments of equal lengths $S_i/sin\theta$ (Fig. 3) where $S_i$ is vertical component of the forces inter-slides, $E_i$ is horizontal component of the forces inter-slides, $W_i$ is unit weight of the slide, $F_i$ is reaction of the embankment inclined at an angle $\varphi$ to the normal on the plan, $T_i$ is tensile strength of the reinforcement, $\theta$ is angle of the segment of failure to the horizontal, $\varphi$ is internal friction angle, $S_v$ is thickness of a section (slide).

To formulate the equilibrium equations corresponding to each of the three mechanical models (slide surfaces) illustrated on Fig. 2, we study the vertical and horizontal balance of an unspecified slide (Fig. 3).

#### A. Plane and circular failure

The mechanical models of plane and circular failures are represented respectively in Figs. 2, a and b. Equilibrium equations of a section of thickness $S_i$ (Fig. 3) can be written:

- projection of forces on vertical

$$W_i + S_i - S_{i+1} - F_i \cos(\theta - \varphi) = 0$$

(2)

- projection of forces on horizontal

$$T_i - E_i + E_{i+1} - F_i \sin(\theta - \varphi) = 0$$

(3)

We make $\Delta E = E_{i+1} - E_i$ and $\Delta S = S_i - S_{i+1}$. From Eq. (2) we have the friction force : $F_i = (W_i + \Delta S) \cos(\theta - \varphi)$, replaced in Eq. (3), we obtain general equilibrium equation for the two slip surfaces cited above

$$T_i + \Delta E - (W_i + \Delta S) \tan(\theta - \varphi) = 0$$

(4)

We can also calculate tensile force in the reinforcement $i$

$$T_i = -\Delta E + (W_i + \Delta S) \tan(\theta - \varphi)$$

(5)

The sum of tensile forces of all the reinforcements cut by the failure surface is in this case equals to:

$$\sum T_i = \sum W_i \tan(\theta - \varphi)$$

(6)

#### B. Mixed failure surface

This model is composed of two failure surfaces: one is inclined at an angle $\theta$ to the horizontal, it starts from the wall footing and ends at the start of the second failure surface, the latter is vertical and extends up to the upper level of the ground (Fig. 2, c). The procedure of formulation of the limit state equation in this case is quite similar to that for circular
and plane surfaces, except that in this case we add earth pressure $P_i$ behind the vertical surface.

The limit equilibrium equation of the mixed failure is written as follows

$$\sum T_i = \sum W_i \tan(\theta - \phi) + P_i \quad \text{(7)}$$

where $P_i$ represents earth pressures behind the vertical failure surface. We have studied in this paper various distributions of the earth pressures (triangular, rectangular, bilinear and elliptic).

The sum of mobilized tensile forces $\sum T_i$ is related to the angle which forms failure surface with the horizontal ($\theta$), the internal friction angle of the ground and the earth pressures (Model of mixed failure).

C. Critical failure surface

We have to search iteratively for each of the three models (Fig. 2), the critical slip surface which gives the maximum mobilized resultant of tensile forces $T_{\text{max}}=\sum T_i$. For that purpose, the above formulated equilibrium equations Eqs. (6) and (7) are programmed by the authors using the Delphi language. The software draws the most critical failure model found by iterative calculations and gives information on the crucial angle of failure $\theta$, the maximum tensile force $T_i$, the resultant of tensile forces $\sum T_i$ and the minimal corresponding safety factor for each model.

To find the most critical mechanical model, we calculate the safety factors against break failure and pullout failure of the reinforcements [3]

$$F_{s_y} = \frac{T_{\text{max}}}{T_{\text{max}}} \quad \text{and} \quad F_{s_f} = \frac{T_{\text{max}}}{T_{\text{max}}} \quad \text{(8)}$$

where $T_{\text{max}}$ is maximum tensile force obtained in the case of a pullout failure of the reinforcement; $T_{\text{max}}$ is maximum tensile force obtained in the case of break failure of the reinforcement.

$$T_{\text{max}} = 2b f^* (\sigma_v + \Delta q) L_e \quad \text{(9)}$$

$$T_{\text{max}} = R b a \quad \text{(10)}$$

where $F_{s_y}, F_{s_f}$ are safety factors, respectively applied to tension strength of the reinforcement and to the lateral friction; $F_{s_y}$ has a value of 1.5 for ordinary constructions and 1.65 for constructions with high safety level; $F_{s_f}$ equals 1.35 for ordinary constructions and 1.5 for constructions with high safety level; $b$ is width of the reinforcement; $a$ is thickness of the reinforcement; $f^*$ is coefficient of apparent friction; $\sigma_v$ is vertical stress; $\Delta q$ is additional stress due to a possible overload; $L_e$ is length of the strip in the zone of resistance; $R$ is failure stress of the reinforcement (metallic strip).

IV. NUMERICAL MODELING

In this research, the software FLAC2d (Fast Lagrangian Analysis of Continua) based on the method of the finite differences was employed to study the internal stability of the reinforced earth walls [8]. It provides the fields of strains and stresses. The continuous medium is discretized in quadrilaterals, each one of them being divided into two pairs of triangular elements with uniform deformation. The failure criterion used in this work is that of Mohr-Coulomb “elastic-plastic”. Boundary conditions are taken into account by blocking horizontal displacement in the direction y and horizontal and vertical displacement for the lower limit (base). The metallic strips are modelled like bars. Input data for the ground, steel strips and interface are summarized in Table 1.

<table>
<thead>
<tr>
<th>Characteristic s of the ground</th>
<th>$\gamma = 1600 , \text{Kg/m}^3$</th>
<th>$\phi = 36^\circ$</th>
<th>$K = 1.6 \times 10^7 , \text{N/m}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Characteristic s of the metallic strip</td>
<td>Number of bands (strips) per unit of width = 1</td>
<td>Width of calculation = 0.75m</td>
<td>Width of the reinforcement = 0.04m</td>
</tr>
<tr>
<td></td>
<td>Thickness of the reinforcement = 0.005m</td>
<td>Modulus of elasticity of the strip = $200 \times 10^7 , \text{N/m}^2$</td>
<td>Maximum tensile force of the strip = 24000 N/m</td>
</tr>
<tr>
<td>Characteristic s of the interface/strip</td>
<td>The rigidity of shearing [N/m/m] = $1 \times 10^7$</td>
<td>Initial coefficient of apparent friction = 1.5</td>
<td>Minimal coefficient of apparent friction = 0.72</td>
</tr>
</tbody>
</table>

| Case 1 : $H = 7.5 \, \text{m}$, $\phi = 36^\circ$ |

Fig. 4 represents the three failure surfaces (plane, circular and mixed) obtained by analytical calculation. The width of the upper part of the failure edge is different in the three models (Fig. 4). The model of mixed failure gives the smallest failure edge, whose highest width is equal to 0.356 $H$ (2.67 m) and the height of vertical failure surface is equal to 0.25 $H$ (1.875 m).
The field of shearing represented in Fig. 5 gives a failure surface which has a form nearer to the model of mixed failure.

Fig. 6 gives the maximum axial force in the reinforcement for each level of the wall for all the models studied. All models give the same results in the vicinity of medium of the wall.

The model of circular failure gives the smallest tensile forces along the wall, except for the lower strip where the model of FLAC gives the smallest force. In general, the model of FLAC gives the greatest tension forces in the strips at all levels. The analytical model of mixed failure with bilinear and elliptic pressures is closer to the FLAC model.

According to Table 2 the model of mixed failure with elliptic pressure gives the smallest safety factors against breaking failure and lack of bond of the reinforcements compared to the other analytical models. This confirms that in this case, the model of mixed failure with elliptic pressure is the most unfavourable.

**Case 2 :** \( H = 6 \text{ m}, \phi = 36^\circ \)

According to the results represented hereafter for a reinforced earth wall of height \( H = 6 \) and for an internal friction angle of \( 36^\circ \) we can note that:

- the maximum tension force in the reinforcement is increasingly larger in the lower part of the wall than that in the upper part (Fig. 7);
- the delimitation of the field of shearing given by Flac2d is closer to the model of mixed failure;
- the curve of the maximum tension forces according to Flac2d gives lower values compared to the other analytical models. The model of mixed failure gives the greatest tension forces in the reinforcements and particularly on the upper level of the wall (Fig. 7);
- according to Table 3 the model of mixed failure with elliptic pressure is the most unfavourable model.

**Case 3 :** \( H = 9 \text{ m}, \phi = 36^\circ \)

The model of mixed failure according to Fig. 8 is the most critical model, with a width of the wedge of 0.33 \( H \) and vertical height of 0.29 \( H \). The model of mixed failure with elliptic pressure gives the greatest tensile forces (Fig. 9).
In conclusion the comparative analysis of the results obtained enabled us to make these important observations:

- The most critical failure model is generally the mixed failure model with elliptic distribution of pressures of the ground behind the vertical failure surface;
- The fields of shearing given by software Flac\textsuperscript{2d} delimit failure contour which is very often close to the mixed failure model;
- The maximum displacement in the reinforced earth walls by failure is obtained at the base of the wall and gives greater tensile forces at this level. Therefore for pre-dimensioning of the reinforcements, we take the force of the last strip by taking account of the results of the tests;
- Height of the wall and internal friction angle have an effect on the geometry of the failure models (slope of the failure surfaces) and tensile forces in the reinforcements;
- The internal friction angle and the height of the wall influence widely the safety factors against breaking failure and lack of bond of the reinforcements. Reduction in the internal friction angle led to a reduction in the two safety factors because of the decrease of the friction ground-strip. The increase of the height of the wall decreases the first and increases the second.

VI. CONCLUSION

In this paper we have studied internal stability of the reinforced earth retaining walls by traditional limit equilibrium method and numerical methods using Flac\textsuperscript{2d} software. The objective of this paper is to find the most dangerous (unfavourable) overall failure model “facing-ground-reinforcement” by reinforced earth retaining walls. A detailed parametric study by varying geometrical parameters of the wall and parameters of the ground (internal friction angle) was carried out. Comparative analysis of the results enabled us to obtain a useful knowledge. The most critical failure model is generally the model of mixed failure. The fields of shearing given by software Flac\textsuperscript{2d} delimit a contour which is very often close to the model of mixed failure.

REFERENCES


