Reliability Analysis in Geotechnical Engineering

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***Abstract*—The evaluation of safety or reliability of the structures with respect to the various risks of instability or catastrophes is done for a long time by empirical methods through the introduction of the total safety factors which are variable from case to another. The role of these factors is to cover uncertainties, risks and the ignorance of the problem, but in an arbitrary way. Consequently, one built stable structures but often non economic. One presents in this paper an new approach based on the probabilistic methods for a good evaluation of the reliability of the structures by taking account of the natural variability (dispersion) of the parameters, of uncertainties, ...etc. one proposes to apply safety factors on every parameter instead of only one global safety factor. The value of the safety factor varies from a parameter to another according to whether the parameter is constant or variable. One must arrange a data base for every parameter through tests and by experience (stochastic analysis). Safety is expressed by means of the reliability index or the probability of failure. Examples of some problems in geotechnical engineering will be given, in order to highlight the utility of this new approach.**

***Keywords*— Probability of failure, reliability, reliability index, risks, safety factors.**

I. Introduction

FOR a long time, we built stable foundations and constructions by increasing the section or the surface of the elements constituting this constructions, and this through the use of arbitrarily given by the experience empirical total safety factors. Consequently, oversize constructions are often obtained. The need for building more reliable and economic constructions led the engineers to develop a new concept of safety based on the theory of probability which should meet these requirements. We use for this purpose the partial factors of safety derived from probabilistic methods to cover the random dispersion of the parameters influencing the stability of the structure instead of a total factor of safety (conventional practical safety approach). The safety of the system is expressed by the probability of failure P_f or the reliability index β defined by Hasofer/Lind [1] and [3].

The verification of the stability of the structures is done traditionally by calculating of a total coefficient of safety F_s = 2 or 3, where F_s is defined classically as the ratio of the strength (capacity) R to the solicitation S.

In this paper, we presented a new analysis of the problem based on the probabilistic theory. By calculations with the probabilistic approach we give up the traditional notion of the total safety factor F_s and we consider R and S as two random variables having each one a mean value and a standard deviation (m_R, σ_R) and (m_S, σ_S). We apply thus partial factors of safety to the solicitations and resistances.

It is well established that input parameters for geotechnical calculations are associated with uncertainties. This holds for material properties as well as for model parameters which have to be introduced when building a geomechanical model, which itself represents only an approximation to the actual situation in situ.

In order to arrive at a probability of "failure", whereas the term "failure" has a very general meaning here as it may indicate collapse of a structure or in a very general form define the loss of serviceability, a limit state function or performance function *G(X)* of the following form can be defined

$$G(X) = R(X) - S(X) \tag{1}$$

R(X) is the "resistance", S(X) is the "action", and X is the collection of random input parameters.

For G(X)<0 failure is implied, while G(X)>0 means stable behaviour. The boundary defined by G(X)=0 separating the stable and unstable state is called the limit state boundary. The probability of failure P_f is defined as:

$$P_f = P\left[G(X) \le 0\right] = \int_{G(X)\le 0} f(X)dx \tag{2}$$

where *f(X)* is the common probability density function of the vector formed by the variables ***X***.

A number of different approaches have been suggested in the literature to integrate (2). In this paper the approach First-order reliability method is used.

Hasofer & Lind (1974) [3] proposed an invariant definition for the reliability index [7]. The approach is referred to as the first-order reliability method (FORM). The starting point for FORM is the definition of the performance function G(X), where X is the vector of basic random variables. In general**,** the above integral cannot be solved analytically. In the FORM approximation, the vector of random variables **X** is transformed to the standard normal space U, where U is a vector of independent Gaussian variables with zero mean and unit standard deviation, and where G(U) is a linear function. The probability of failure P_f is then (P[…] means probability that …):

$$P_f = P[G(U) < 0] \approx P\left[\sum_{i=1}^{n} \alpha_i . U_i - \beta < 0\right] = \phi(-\beta) \qquad (3)$$

where α_i is the direction cosine of random variable U_i, β is the distance between the origin and the hyperplane G(U) = 0, n is the number of basic random variables X, and ϕ is the standard normal distribution function.

The vector of the direction cosines of the random variables (α_i) is called the vector of sensitivity factors, and the distance β is the reliability index. The probability of failure (P_f) can be estimated from the reliability index β using the established equation $P_f = 1 - \phi(\beta) = \phi(-\beta)$, where ϕ is the cumulative distribution (CDF) of the standard normal variate. The relationship is exact when the limit state surface is planar and the parameters follow normal distributions, and approximate otherwise. The relationship between the reliability index and probability of failure defined by (3) is shown in Table 1.

Table 1: The relationship between the reliability Index and probability of failure.

β [-]	5.2	4.7	4.2	3	2.5	2.0
P_f [-]	$\approx 10^{-7}$	$\approx 10^{-6}$	$\approx 10^{-5}$	$\approx 10^{-3}$	$\approx 5.10^{-3}$	$\approx 10^{-2}$

The square of the direction cosines or sensitivity factors (α_i^2), whose sum is equal to unity, quantifies in a relative manner the contribution of the uncertainty in each random variable X_i to the total uncertainty.

In summary the FORM approximation involves:
1. Transforming a general random vector into a standard Gaussian vector,
2. Locating the point of maximum probability density (most likely failure point, design point, or simply β-point) within the failure domain, and
3. Estimating the probability of failure as $P_f = \phi(-\beta)$, in which ϕ(-) is the standard Gaussian cumulative distribution function.

An illustration of the design point and graphical representation of β is given in Figure 1.

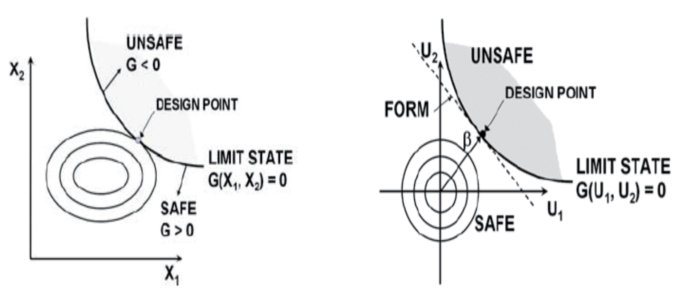

Figure 1: The FORM approximation and definition of β and design point [7].

II. Advanced Second Moment (ASM) Method

The reliability of a structure can be determined based on a performance function that can be expressed in terms of basic random variables *Xi*'s for relevant loads and structural strength. Mathematically, the performance function *Z* can be described as [8]:

$Z = Z(X_1,\ X_2, \ldots\ldots X_3)$ = *Structural strength - Load effect* (4)

where *Z* is called the *performance function* of interest. The unsatisfactory performance surface (or the *limit state*) of interest can be defined as *Z* = 0. Accordingly, when *Z* < 0, the structure is in the unsatisfactory performance state, and when *Z* > 0, it is in the safe state. If the joint probability density function for the basic random variables X_i 's is $f_{x_1\ x_2 \ldots\ldots x_n}(x_1, x_2, \ldots, x_n)$, then the unsatisfactory performance probability P_f of a structure can be given by the integral

$$P_f = \int \ldots \int f_{x_1, x_2, \ldots, x_n}(x_1, x_2, \ldots\ldots\ldots x_n) dx_1 dx_2 \ldots\ldots\ldots dx_n \qquad (5)$$

where the integration is performed over the region in which Z <0. In general, the joint probability density function is unknown, and the integral is a formidable task. For practical purposes, alternate methods of evaluating P_f are necessary [8].

II.A Reliability index

Instead of using direct integration as given by (5), the performance function *Z* in (4) can be expanded using a Taylor series about the mean value of *X*'s and then truncated at the linear terms [8]. Therefore, the first-order approximate mean and variance of *Z* can be shown, respectively, as

$$\mu_z \cong Z\left(\mu_{X_1}, \mu_{X_2}, \ldots\ldots\ldots, \mu_{X_n}\right) \qquad (6)$$

and

$$\sigma_Z^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{\partial Z}{\partial X_i}\right)\left(\frac{\partial Z}{\partial X_j}\right).\mathrm{cov}\left(X_i, X_j\right) \qquad (7)$$

where
μ_Z = mean of *Z*
μ = mean of a random variable
σ_Z^2 = variance of *Z*
$Cov(X_i, X_j)$ = the covariance of X_i and X_j

The partial derivatives of $\partial Z/\partial X_i$ are evaluated at the mean values of the basic random variables. For statistically independent random variables, the variance xpression can be simplified as

$$\sigma_Z^2 = \sum_{i=1}^{n}\sigma_{X_i}^2\left(\frac{\partial Z}{\partial X_i}\right)^2 \qquad (8)$$

A measure of reliability can be estimated by introducing the *reliability index β* that is based on the mean and standard deviation of *Z* as

$$\beta = \frac{\mu_Z}{\sigma_Z} \qquad (9)$$

The aforementioned procedure produces accurate results when the performance function Z is normally distributed and linear.

II.B Nonlinear performance functions

For nonlinear performance functions, the Taylor series expansion of Z is linearized at some point on the unsatisfactory performance surface called *design point*, *checking point*, or *the most likely unsatisfactory performance point* rather than at the mean [8]. Assuming the original basic variables X_i 's are uncorrelated, the following transformation can be used:

$$Y_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}} \quad (10)$$

If Xi 's are correlated, they need to be transformed to uncorrelated random variables. The reliability index β is defined as the shortest distance to the unsatisfactory performance surface from the origin in the reduced Y-coordinate system. The point on the unsatisfactory performance surface that corresponds to the shortest distance is the most likely unsatisfactory performance point. Using the original X-coordinate system, the reliability index β and design point (X_1*, X_2*,……., X_n*) can be determined by solving the following system of nonlinear equations iteratively for β:

$$\alpha_i = \frac{\left(\frac{\partial Z}{\partial X_i}\right).\sigma_{X_i}}{\left[\sum_{i=1}^{n}\left(\frac{\partial Z}{\partial X_i}\right)^2 .\sigma_{X_i}^2\right]^{1/2}} \quad (11)$$

$$X_i^* = \mu_{X_i} - \alpha_i.\beta.\sigma_{X_i} \quad (12)$$

$$Z\left(X_1^*, X_2^*, ..., X_n^*\right) = 0 \quad (13)$$

where α_i is a directional cosine and the partial directives are evaluated at the design point, indicated by an asterisk. However, this formulation is limited to normally distributed random variables. In reliability assessment, the directional cosines can be viewed as measures of the importance of the corresponding random variables in determining the reliability index β. Also, partial safety factors γ that are used in load and resistance factor design can be computed as follows:

$$\gamma = \frac{X^*}{\mu_X} \quad (14)$$

In general, partial safety factors take on values larger than one for load variables, and values less than one for strength variables.

II.C Equivalent normal distributions

If a random variable X is not normally distributed, then it needs to be transformed to an equivalent normally distributed random variable, indicated by the superscript N. The parameters of the equivalent normal distribution, $\mu_{X_i}^N$ and $\sigma_{X_i}^N$, can be estimated by imposing two conditions [6]. The cumulative distribution functions (CDF) and probability density functions of a nonnormal random variable and its equivalent normal variable should be equal at the design point on the unsatisfactory performance surface. The first condition can be expressed as

$$\Phi\left(\frac{X_i^* - \mu_{X_i}^N}{\sigma_{X_i}^N}\right) = F_i\left(X_i^*\right) \quad (15)$$

The second condition is

$$\phi\left(\frac{X_i^* - \mu_{X_i}^N}{\sigma_{X_i}^N}\right) = f_i\left(X_i^*\right) \quad (16)$$

where

F_i = nonnormal cumulative distribution function

ϕ = probability density function of standard normal variate

f_i = nonnormal probability density function

The standard deviation and mean of equivalent normal distributions can be shown, respectively, to be

$$\sigma_{X_i}^N = \frac{\phi\left\{\Phi^{-1}\left[F_i\left(X_i^*\right)\right]\right\}}{f_i\left(X_i^*\right)} \quad (17)$$

and

$$\mu_{X_i}^N = X_i^* - \Phi^{-1}\left[F_i\left(X_i^*\right)\right]\sigma_{X_i}^N \quad (18)$$

Having determined $\mu_{X_i}^N$ and $\sigma_{X_i}^N$ for each random variable, β can be solved using the same procedure of Equations 11-13.

II.D Correlated random variables

Reliability analysis of gravity structures needs to be based on correlated soil properties such as angle of internal friction and cohesion for soil layers. In this section, this correlation is assumed to occur between pairs of random variables for each layer. Also, correlated random variables are assumed to be normally distributed since nonnormal and correlated random variables require additional information such as their joint probability density function or conditional distributions for their unique and full definition [8]. Such information is commonly not available and difficult to obtain. A correlated (and normal) pair of random variables X_1 and X_2 with a correlation coefficient ρ can be transformed into noncorrelated pair Y_1 and Y_2 by solving for two eigenvalues λ and the corresponding eigenvectors as follows:

$$Y_1 = \frac{1}{2\sqrt{0,5}}\left(\frac{X_1 - \mu_{X_1}}{\sigma_{X_1}} + \frac{X_2 - \mu_{X_2}}{\sigma_{X_2}}\right) \quad (19)$$

$$Y_2 = \frac{1}{2\sqrt{0,5}}\left(\frac{X_1 - \mu_{X_1}}{\sigma_{X_1}} - \frac{X_2 - \mu_{X_2}}{\sigma_{X_2}}\right) \quad (20)$$

The resulting Y variables are noncorrelated with respective variances that are equal to the eigenvalues λ as follows:

$$\sigma_{Y_1}^2 = \lambda_1 = 1 + \rho \tag{21}$$

$$\sigma_{Y_2}^2 = \lambda_2 = 1 - \rho \tag{22}$$

For a correlated pair of random variables, Equations 11 and 12 need to be revised respectively to the following:

$$\alpha_{Y_1} = \frac{\left[\left(\frac{\partial Z}{\partial X_1}\right).\sqrt{0{,}5}.\sigma_{X_1} + \left(\frac{\partial Z}{\partial X_2}\right).\sqrt{0{,}5}.\sigma_{X_2}\right].\sqrt{1+\rho}}{\left[\left(\frac{\partial Z}{\partial X_1}\right)^2.\sigma_{X_1}^2 + \left(\frac{\partial Z}{\partial X_2}\right)^2.\sigma_{X_2}^2 + 2.\rho.\left(\frac{\partial Z}{\partial X_1}\right)\left(\frac{\partial Z}{\partial X_2}\right).\sigma_{X_1}.\sigma_{X_2}\right]^{1/2}} \tag{23}$$

$$\alpha_{Y_2} = \frac{\left[\left(\frac{\partial Z}{\partial X_1}\right).\sqrt{0{,}5}.\sigma_{X_1} - \left(\frac{\partial Z}{\partial X_2}\right).\sqrt{0{,}5}.\sigma_{X_2}\right].\sqrt{1-\rho}}{\left[\left(\frac{\partial Z}{\partial X_1}\right)^2.\sigma_{X_1}^2 + \left(\frac{\partial Z}{\partial X_2}\right)^2.\sigma_{X_2}^2 + 2.\rho.\left(\frac{\partial Z}{\partial X_1}\right)\left(\frac{\partial Z}{\partial X_2}\right).\sigma_{X_1}.\sigma_{X_2}\right]^{1/2}} \tag{24}$$

and

$$X_1^* = \mu_{X_1} - \sigma_{X_1}.\sqrt{0{,}5}.\beta.\left(\alpha_{Y_1}.\sqrt{\lambda_1} + \alpha_{Y_2}.\sqrt{\lambda_2}\right) \tag{25}$$

$$X_2^* = \mu_{X_2} - \sigma_{X_2}.\sqrt{0{,}5}.\beta.\left(\alpha_{Y_1}.\sqrt{\lambda_1} - \alpha_{Y_2}.\sqrt{\lambda_2}\right) \tag{26}$$

where the partial derivatives are evaluated at the design point.

II.E Numerical algorithms

The ASM method can be used to assess the reliability of a structure according to a nonlinear performance function that may include nonnormal random variables. Also, the performance function can be in a closed or nonclosed form expression. The implementation of this method requires the use of efficient and accurate numerical algorithms in order to deal with the nonclosed forms for performance function. The ASM algorithm can be summarized by the following steps using two cases [8]:

Algorithm 1 (noncorrelated random variables)

Use the following steps:

(1) Assign the mean value for each random variable as a starting design point value, i.e.,
$(X_1^*, X_2^*, \ldots, X_n^*) = (\mu_{X1}, \mu_{X2}, \ldots, \mu_{Xn})$.

(2) Compute the standard deviation and mean of the equivalent normal distribution for each nonnormal random variable using Equations 15-18.

(3) Compute the partial derivative $\partial Z/\partial X_i$ of the performance function with respect to each random variable evaluated at the design point as needed by Equation 11.

(4) Compute the directional cosine α_i for each random variable as given in Equation 11 at the design point.

(5) Compute the reliability index β by substituting Equation 12 into Equation 13 and satisfying the limit state $Z = 0$ in Equation 13 using a numerical root-finding method.

(6) Compute a new estimate of the design point by substituting the resulting reliability index β obtained in Step 5 into Equation 12.

(7) Repeat Steps 2 to 6 until the reliability index β converges within an acceptable tolerance.

Algorithm 2 (correlated random variables)

Use the following steps:

(1) Assign the mean value for each random variable as a starting design point value, i.e.,
$(X_1^*, X_2^*, \ldots, X_n^*) = (\mu_{X_1}, \mu_{X_2}, \ldots, \mu_{X_n})$.

(2) Compute the standard deviation and mean of the equivalent normal distribution for each nonnormal random variable using Equations 15-18.

(3) Compute the partial derivative $\partial Z/\partial X_i$ of the performance function with respect to each random variable evaluated at the design point as needed by Equation 11.

(4) Compute the directional cosine α_i for each noncorrelated random variable as given in Equation 11 at the design point. For correlated pairs of random variables, Equations 23 and 24 should be used.

(5) Compute the reliability index β by substituting Equations 12 (for noncorrelated random variables) and 23 and 24 (for correlated random variables) into Equation 11 and satisfying the limit state $Z = 0$ in Equation 11 using a numerical root-finding method.

(6) Compute a new estimate of the design point by substituting the resulting reliability index β obtained in Step 5 into Equations 10 (for noncorrelated random variables) and 25 and 26 (for correlated random variables).

(7) Repeat Steps 2 to 6 until the reliability index β converges within an acceptable tolerance.

III. EXAMPLES

In this paper, the computation of the reliability index is done by a software based on the first order reliability method [1] by using the algorithm 1.

Table 2: Statistical Data of the Random Variables [1].

Parameter	Type of distribution	Coefficient of variation	Characteristic value
Internal friction angle φ' [°]	Log-normal	10 %	20 to 40
Cohesion c' [kPa]	Log-normal	25 %	0 to 35
Volumetric weight of soil γ [kN/m^3]	Normal	5 %	20
Permanent external load Q [kN/ml]	Normal	10 %	290 to 800
Inclination of the load δ [°]	Normal	10 %	10

Table 2 summarizes the random variables and their statistical properties necessary for these calculations.

We neglect in this paper the effect of the correlation between variables and of the autocorrelation and we suppose that the characteristic values of the parameters were given with prudence in accordance with the recommendations of Eurocode 7 [5]. The geometrical parameters present in general a negligible dispersion compared to that of the shear parameters of the soil and of the loads. This is why we consider them as deterministic parameters.

III.A Failure of the shallow foundations

The verification of the stability of the shallow foundations with respect to punching is done traditionally by applying a total coefficient of safety F_s = 2 or 3 on the limit load calculated by the theory of Prandtl. In this paper, we presented a new analysis of the problem based on the probabilistic theory (Eurocode 7) by applying partial factors of safety to forces (Approach 2) or to soil parameters (Approach 1). A comparative study with the national traditional practical approaches (deterministic approaches) of France (DTU 13.12) [4] and Germany (DIN 1054) [2] was carried out.

Parametric studies were carried out for a foundation of width B and length L having an embedment D=1.50m in a homogeneous soil (Table 2). The volumetric weight of the concrete is taken equal to 24 kN/m^3.

The results of computations of the reliability indices for some cases are given in Figures 2 and 3 for various types of grounds according to various approaches. For more details and cases see [1].

The interpretation of these results enabled us to make the following observations:

- French traditional practical approach DTU 13.12 (F_s=3) and approach 3 (EC. 7) underestimate safety by giving the greatest widths of foundations and the greatest indices of reliability. The corresponding security level is inhomogeneous for all the studied soils (Fig. 2).
- security level obtained for approach 1 (EC. 7), whose partial factors of safety are applied to the soil parameters, is relatively homogeneous along the types of the studied soils in comparison with the approaches whose safety factors are applied to forces (Approaches 2 (EC. 7), DIN 1054 (F_s=2) and DTU 13.12 (F_s=2)). These last approaches present the same tendency in the values of β (Figs. 2 and 3). The values of β obtained by approach 1 are close to the target values β =3.
- In all the studied cases, the ultimate limit state approaches of Eurocode 7 (Approaches 1 and 2) are most unfavourable by far. They give the minimum of safety and smallest dimensions of the foundations. Approach 2 is most unfavourable for non-cohesive soils and soils with low cohesion (c'≤10 kPa). Approach 1 (combination 2) is most unfavourable for the soils with medium and high cohesion (c'>10 kPa).

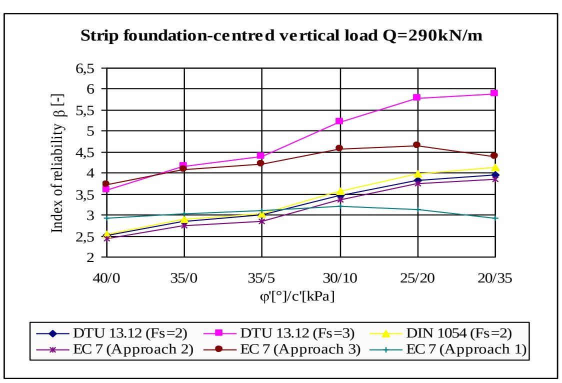

Figure 2: Reliability index β [-]according to various approaches.

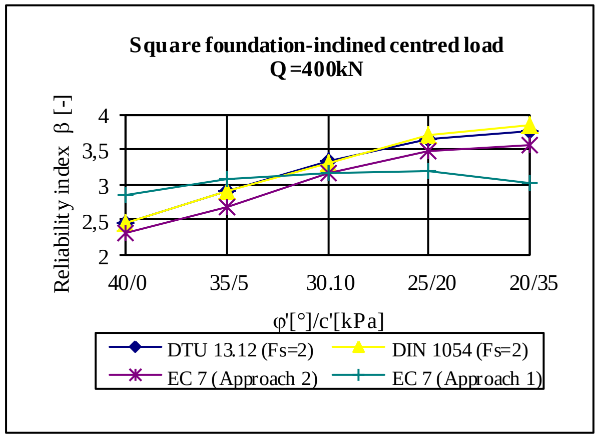

Figure 3: Reliability index β [-] according to various approaches.

III.B Modelling of failure mechanism for anchored retaining walls

In order to determine the overall stability and the necessary anchor lengths of anchored retaining walls, the failure in the deep slip surface is often investigated. In this study a comparison between the kinematical model nearest to reality which is based on the kinematical theory of rigid bodies and the simplified model is carried out. By the simplified model, a fictitious vertical anchor wall is placed at the intersection point of the deep slip surface with the injection anchor. By the kinematical model, an inclined internal slip surface is placed at the intersection point of the deep slip surface with the injection anchor. This is the exclusive difference between both models. The comparison is done following the probabilistic safety approach. System safety is estimated by the safety or reliability index.

The anchor forces (A_1 and A_2) act on the rear active slip body number 2 because the corresponding anchors are intersected once from the internal slip surface (Fig. 4). Thus, the intersected anchors prevent partly soil loosening.

The limit state equation of the kinematical model is:

$$\begin{aligned}&E_p\cos(\delta_p-\theta_3+\varphi)+C_1\cos\varphi+C_{12}\cos(\theta_{12}+\theta_3-\varphi)+\\&+(A_1+A_2)\cos(\theta_3-\varphi+\varepsilon)+C_2 m_3\cos\varphi-(G_1+P_1)\sin(\theta_3-\varphi)-\\&-[(G_2+P_2)\sin(\theta_{11}-\varphi)+(A_1+A_2)\cos(\theta_{11}-\varphi+\varepsilon)].m_3\end{aligned} \qquad (27)$$

in which $m_3=\dfrac{\sin(\theta_{12}+\theta_3-2\varphi)}{\sin(\theta_{12}+\theta_{11}-2\varphi)}$

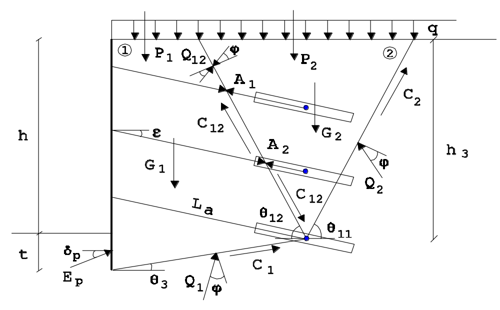

Figure 4: Kinematical model by the failure of the lower anchor.

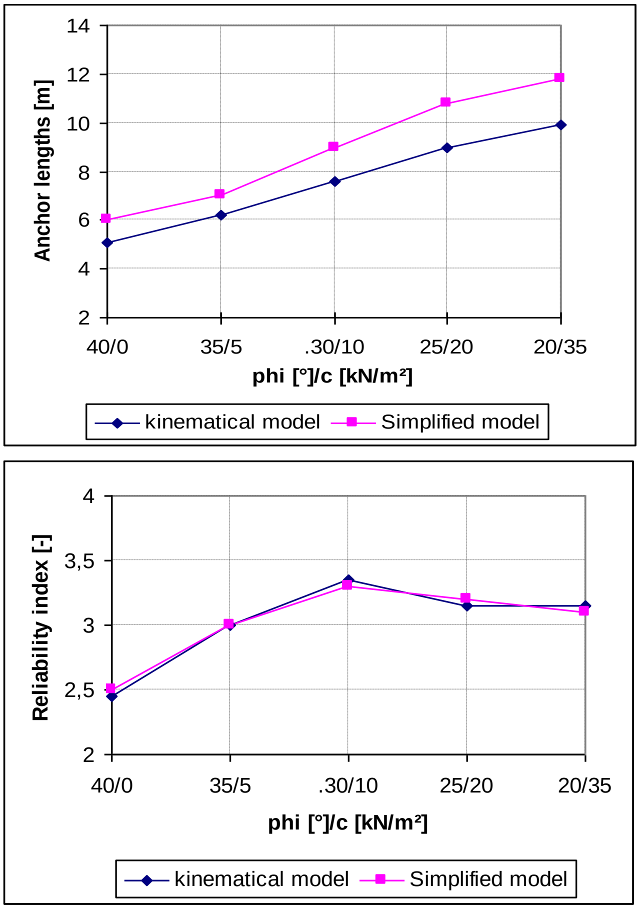

Figure 5: Comparison of the kinematical and simplified models.

The approach with the fictitious anchor wall is misleading because the effect of intersected anchor forces is neglected. The results are presented in Fig. 5 which shows in detail the courses of anchor lengths and reliability level (ß-level) for that concerned failure model. By the failure of the lower anchor, the simplified model yields longer anchor lengths than the kinematical model. The difference is clearly larger (approximately 20%) (Fig. 5). Nevertheless, both models yield equal reliability indexes ß. In this case it is to advise, the simplified model not instead of the kinematical model to investigate since the simplified model underestimates here the system safety (reliability).

IV. Conclusion

The verification of the stability of the structures is done traditionally by calculating of a total coefficient of safety which is defined classically as the ratio of the strength (capacity) to the solicitation and is variable from case to another. In this paper, one presents a new analysis of the problem based on the probabilistic theory. One proposes to apply partial safety factors on every parameter instead of only one global safety factor. The value of the partial safety factor varies from a parameter to another according to whether the parameter is constant or variable. The probabilistic approach makes it possible to make an objective comparison of the various types of risks what was not possible with the old deterministic approach. The probabilistic approach is a means to standardize the codes and the standards treating the calculation of the structures by replacing the empirical total safety factors by partial safety factors taking account of the random dispersion of the parameters. Moreover, this approach ensures an often economic use of materials.

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