Natural Convection Heat Transfer in a Differentially Heated Square Enclosure with a Heat Generating-Conducting Circular Cylinder at Different Diagonal Locations

Salam Hadi Hussain and Ahmed Kadhim Hussein*

1University of Babylon, Babylon Province/ Iraq, salamphd1974@yahoo.com
2University of Babylon, Babylon Province/ Iraq, ahmedkadhim74@yahoo.com

Abstract—The present research deals with steady free convection heat transfer in a differentially heated square enclosure filled with air with an interior heat generating conducting solid circular cylinder at different diagonal locations. The purpose of the present study is to examine how the location of the interior cylinder and its heat generation affect on the free convection phenomena for various Rayleigh numbers when an interior cylinder moves at various locations along the diagonal of the outer square enclosure. The steady governing equations are solved numerically by using the finite volume approach. The present work results explain that increase in the heat generation and Rayleigh numbers have a clear effect on the stream and isotherm contours. While, the interior cylinder locations do not play an important role when the interior cylinder does not generate heat. Also, the results show that the average Nusselt number at the hot side wall decreases for all values of (δ) when the heat generation value increases, while the average Nusselt number at the cold side wall increases for all values of (δ) when the heat generation value and Rayleigh number increases. A comparison of the results showed a good agreement with another published results.

Keywords—Natural convection, heat generating, circular cylinder, square enclosure, conducting, diagonal locations, finite volume.

I. INTRODUCTION

Free convective and fluid flow from a heated body, especially cylinder, inside enclosures has long been studied and has received more attention due to its direct relevancy to many engineering applications. As one of the representative geometries, laminar free convection around a conductive circular cylinder embedded inside a square enclosure is of great interest since such geometry is commonly applied to represent cooling of electronic equipments, nuclear and chemical reactors, flooding protection for buried pipes, solidification processes, growing crystals and solar collectors. House et al. [1], investigated the effect of a centered, conducting body on natural convection heat transfer in a vertical enclosure. They concluded that heat transfer process across the enclosure may be increased or decreased by a conducting body with a thermal conductivity ratio lower or greater than unity. Oh et al. [2], used a numerical simulation to study natural convection in a vertical square enclosure containing heat generating conducting body, when a temperature difference existed across the enclosure. The streamlines, isotherms and average Nusselt number at the hot and cold walls are presented and discussed. Roychowdhury et al. [3], analyzed the natural convective flow and heat transfer features for a heated cylinder kept in a square enclosure with different thermal boundary conditions. Braga and De Lemos [4], performed a numerical investigation of steady laminar natural convection within a square cavity filled with a fixed volume of conducting solid material consisting of either circular or square obstacles. They observed that the average Nusselt number for cylindrical rods was slightly lower than those for square rods. Mezrhab et al. [5], presented a numerical investigation of the radiation-natural convection interactions in a differentially heated square cavity within which a centered, square, heat-conducting body generates heat. They concluded that the streamlines and isotherms are greatly affected by the radiation exchange at high Rayleigh numbers. Arnab et al. [6], performed an extensive analysis of flow pattern and heat transfer from a heated rectangular cylinder enclosed inside an enclosure with three different aspect ratio. The effect of aspect ratio and two kinds of boundary conditions (i.e. constant wall heat flux and constant wall temperature) were studied. They concluded that the uniform wall temperature heating was greatly different from the uniform wall flux heating. Gomez [7], simulated 2D and 3D natural convection and radiation heat transfer in a cylinder within a square enclosure in order to observe the differences between the different models and to see how the presence of radiation affected the transport of energy in the enclosure. Results showed that simplified 2D models can be used with a little error in results and that the 2D simulations tend to over predict resulting temperatures by as much as 5% for the higher heat loads when compared to 3D models. Jami et al. [8], analyzed natural convection heat transfer in a differentially heated enclosure, within which a centered, circular, heat-conducting body generates heat. They concluded that for a constant Rayleigh number, the average Nusselt number at the hot and cold walls varied

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linearly with temperature-difference ratio. Angeli et al. [9], presented important results related to natural convection heat transfer from a horizontal cylinder centred in an air filled cavity of square cross-section. A correlating equation for the average Nusselt number on the cylinder, as a function on both the Rayleigh number and the diameter-to-side ratio, was derived, covering the whole steady-state region. Sidik and Abdul Rahman [10], studied numerically, the fluid flow behavior and heat transfer mechanism from a heated square cylinder located at various heights inside a square enclosure by the mesoscale numerical method in the range of $10^3 \leq \text{Ra} \leq 10^6$. Their computational results proved that the flow pattern, number, size and formation of vortices and also heat transfer mechanism were critically dependence on Rayleigh number and the position of heated inner square cylinder in enclosure. Lee et al. [11], studied numerically, two-dimensional natural convection induced by a temperature difference between a cold outer square cylinder and a hot inner circular cylinder. The location of the inner circular cylinder was changed horizontally along the centerline of square enclosure or diagonally along a diagonal line of the square enclosure. The existence of local peaks of the Nusselt number along the surfaces of the cylinder and the enclosure was determined by the gap and the thermal plume governed by the conduction and the convection, respectively. Hussain and Hussein[12], considered the problem of natural convection in a square enclosure which had isothermal walls and was heated by a concentric internal circular isoflux boundary. They concluded that vortices down the bottom and up the top of the inner cylinder can be noticed as the inner cylinder moved upward and downward. However, a very little literatures deal with natural convection problem when a conductive circular cylinder with heat generation embedded inside a square enclosure and moves at different locations along the enclosure diagonal. The objective of this study is to simulate two-dimensional free convection heat transfer in a cylinder within a square enclosure in order to investigate the effect of a hot conductive circular cylinder location and its heat generation on the heat transfer and fluid flow in an enclosure.

II. GEOMETRY DESCRIPTION AND MATHEMATICAL FORMULATION

Consider a circular conductive solid cylinder embedded inside a square enclosure filled with air as a working fluid ($\text{Pr} = 0.71$), where its height and width (L) are considered the same. Both the circular conductive solid cylinder and a square enclosure are subjected to steady free convection of a Newtonian fluid, considering the effect of viscous dissipation is negligible. The fluid flow is two-dimensional and all fluid properties are considered constant except for the density variation in the buoyancy term which is treated according to Boussinesq approximation. The problem geometry is shown in Fig. 1. In the square enclosure, an isothermal hot temperature $T_h$ is applied at the left side wall, while an isothermal cold temperature $T_c$ is applied at the right one. The other walls of an enclosure are considered insulated. With respect to interior circular cylinder, it has radius $R = 0.2L$ and thermal conductivity, $(k_s)$ and it moves along the enclosure diagonal in the range from $-0.25L$ to $+0.25L$ with a step of 0.05L. The interior circular solid cylinder generates heat and four different levels of heat generation ($G$) are taken in account which are (0, 10, 50, 100) while the solid-fluid thermal conductivity ratio is considered constant at 5. The Rayleigh number, Ra, varies in the range from $10^3$ to $10^6$. The two-dimensional flow and thermal fields are governed by the following non-dimensional equations:

$$
\frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = 0 \tag{1}
$$

$$
\frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = - \frac{\partial P}{\partial x} + Pr \left( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right) \tag{2}
$$

$$
\frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} = - \frac{\partial P}{\partial y} + Pr \left( \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right) + Ra Pr \theta \tag{3}
$$

$$
\frac{\partial \theta}{\partial x} + V \frac{\partial \theta}{\partial y} = \left( \frac{\partial^2 \theta}{\partial x^2} \right) \tag{4}
$$

while, the energy equation related to the heat generation interior circular solid cylinder is given by:

$$
\left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + G \right) = 0 \tag{5}
$$

The above equations are converted into a non-dimensional form under the following dimensionless quantities:

$$
\theta = \frac{T - T_c}{T_h - T_c} , x = \frac{x}{L} , y = \frac{y}{L} , U = \frac{uL}{\alpha} , V = \frac{vL}{\alpha} , P = \frac{\rho L^2}{\rho \sigma} \tag{6}
$$

Figure 1: Schematic diagram of the square enclosure with inner heat generating-conducting circular solid cylinder along with coordinate system and boundary conditions.
Natural Convection Heat Transfer in a Differentially Heated Square Enclosure with ...

\[ Pr = \frac{v}{\alpha} \quad G = \frac{q_i A_i}{r_s (T_h - T_c)} \quad \text{and} \quad Ra = \frac{g \beta (T_h - T_c) L^3 P r}{v^2} \]  

(5)

where all the symbols are defined in detail in the Nomenclature. The average Nusselt number, \( N_u_{av\,\text{enc}} \), at the hot and cold side walls (i.e., at \( X=0 \) and \( X=1 \)) is given by [13]:

\[ N_u_{av\,\text{enc}} = \int_0^1 \frac{\partial \theta}{\partial X} \bigg|_{X=0} \, dY \]  

(7)

Boundary conditions: The present problem is solved under the following non-dimensional boundary conditions:

1. The left side wall of the square enclosure is subjected to an isothermal hot temperature, so:
   - at \( X=0 \) \( \theta = 1 \) and \( U = V = 0 \)
2. The right side wall of the square enclosure is subjected to an isothermal cold temperature, so:
   - at \( X=1 \) \( \theta = 0 \) and \( U = V = 0 \)
3. The upper and lower walls of the square enclosure are considered to be insulated so:
   - at \( Y=0 \) and \( Y=1 \) \( \frac{\partial \theta}{\partial Y} = 0 \)
4. At the vertical solid – fluid interfaces:
   - \( \frac{\partial \theta}{\partial X} \) fluid = \( K \frac{\partial \theta}{\partial X} \) solid
5. At the horizontal solid – fluid interfaces:
   - \( \frac{\partial \theta}{\partial Y} \) fluid = \( K \frac{\partial \theta}{\partial Y} \) solid

III. NUMERICAL APPROACH AND CODE VERIFICATION

The finite volume formulation described by Ciarlet and Lions [14] is adopted to discretize the non-dimensional governing equations. The solution is obtained by solving the non-dimensional governing equations simultaneously on a collocated grid system. A Cartesian coordinate is used with origin at the lower left corner of the computational domain, and as a result a FORTRAN program code is developed for this purpose. Fig.2 explains grid system (200 x 100) with non-orthogonal and non-uniform distributions for \( \delta = 0 \). The solid (cylinder) and fluid (air) zones are solved simultaneously by introducing a block parameter, which distinguishes the solid zone from the fluid zone. In order to select the appropriate mesh, the effect of number of grid points on the flow field parameters is examined. For example, a square enclosure with the horizontal conductive circular cylinder at \( Ra=10^6, \text{Pr} = 0.71, \text{G} = 100, K=5 \) and \( \delta = 0.25 \) is selected. In the current work, eight combinations (70 x 70, 80 x 80, 100 x 80, 100 x 100, 120 x 120, 200 x 100, 200 x 200 and 300 x 300) of non-uniform grids are utilized to make the required test. It is observed that, at grid sizes greater than (120 x 120), the variation of average Nusselt number at the hot wall (\( N_u_{avh} \)) appears to be negligible as shown in Fig.3. This suggests that to prevent excessive computational time, a grid size of (120 x 120) is selected for the present work. To begin with the numerical solution, the average Nusselt number values obtained by the present code are validated against values obtained by Kim et al. [15] and Moukalled and Acharya [16]. A very good agreement is achieved during the comparison as shown in Table 1. After that, the code is used to examine the present problem. Finally, the convergence criterion of all the governing equations is taken to be less than \( 10^{-7} \).

IV. RESULTS AND DISCUSSION

Figures 4 and 5 represent the numerical results related to stream lines contours (right) and isotherms (left) at various values of dimensionless heat generation (\( G \)) and the location of the interior cylinder along the diagonal of the square enclosure (\( \delta \)) for Rayleigh numbers \( 10^3 \) and \( 10^6 \) respectively as a study cases. For both figures, the solid-fluid thermal conductivity ratio (\( K \)) is considered constant at 5. As shown in the figures, four different levels of heat generation (\( G \)) are taken in account, which are \( (0, 10, 50, 100) \). When \( Ra = 10^3 \) and for case of interior cylinder with no heat generation (\( G = 0 \)), the isotherms contour are in general take the shape of straight lines which are parallel to the enclosure side walls. This behaviour can be observed, when the interior circular conductive solid cylinder moves upward along the enclosure diagonal from \( (\delta = 0) \) to \( (\delta = 0.25) \) or it moves downward along the enclosure diagonal from \( (\delta = 0) \) to \( (\delta = -0.25) \) which indicating that the heat is transferred due to conduction. In this case, the free convection and buoyancy force effects are in general slight.

![Figure 2: A typical grid distribution (200 x 100) with non-uniform and non-orthogonal distributions for \( \delta = 0 \).](image)

![Figure 3: Convergence of average Nusselt number along the heated left wall of the square enclosure with grid refinement for \( Ra = 10^6, \text{Pr} = 0.71, G = 100, K=5 \) and \( \delta = +0.25 \).](image)
From the other hand, the streamline contours, show an identical rotating vortices around the interior circular conductive cylinder. This similarity between the streamline contours is very clear when the interior circular conductive cylinder locates at the enclosure diagonal center (i.e., $\delta = 0$). Similar behavior can be detected at $\delta = +0.05$ and $\delta = -0.05$ respectively. This results lead to conclude that a slight influence of $\delta$ can be shown on the flow and thermal fields for the small range of $\delta$ from $0$ to $+0.05$ and from $0$ to $-0.05$ respectively. When the interior circular conductive cylinder moves along the enclosure diagonal upward until $\delta = +0.1$, or when moves downward until $\delta = -0.1$, a very small minor vortices can be observed down or up the interior cylinder respectively. As the interior cylinder, precedes continuously along the enclosure diagonal upward from $\delta = +0.15$ to $\delta = +0.25$, or downward from $\delta = -0.15$ to $\delta = -0.25$, the minor vortices begin to enlarge down or up the interior cylinder respectively and occupy most of the enclosure size. This phenomena indicates that the flow field behavior changes clearly as a result of interior circular conductive cylinder movement, while the isotherms become almost the same, when the Rayleigh number is small (i.e., $Ra = 10^3$) and with no heat generation ($G = 0$). When the heat generation values of interior cylinder increase to 10, 50 and 100, a significant deviation and disturbance can be observed in the isotherms and streamlines. In this case, the interior cylinder begins to generate heat and as a result a small hot fluid region can be noticed around the interior cylinder. The thickness of this hot fluid region around the interior cylinder increases as the heat generation values increase. When the heat generation increase to $G=100$, a high temperature gradient can be detected around the interior cylinder. This is because at a high value of the heat generation, the hot air temperature becomes greater than the cylinder surface temperature. In general, for various values of the location of the interior cylinder along the enclosure diagonal ($\delta$), the isotherm begins to change its shape and deviates significantly from parallel straight lines to highly curved non-uniform lines. Moreover, a thermal boundary layer can be observed around the interior cylinder and near the enclosure left and right side walls, and its thickness increases as the heat generation increases. From the other hand, the heat generated by the interior cylinder influences the shapes of the stream contours. In this case, the flow circulation intensity increases and the flow field around the interior cylinder is divided into two large vortices. These vortices are approximately symmetrical when the interior cylinder moves along the enclosure diagonal upward from $\delta = 0$ to $\delta = +0.05$, or downward from $\delta = 0$ to $\delta = -0.05$. After that, as the interior cylinder moves along the enclosure diagonal upward from $\delta = +0.1$ to $\delta = +0.25$, or downward from $\delta = -0.1$ to $\delta = -0.25$, the vortices which lie down or up the interior cylinder are increase in size respectively. When the Rayleigh number increases to $Ra = 10^6$, the flow vortices circulation increases dramatically and becomes more stronger, which leads to make the free convection and buoyancy effects more dominant for different values of heat generation and the interior cylinder along the enclosure diagonal locations ($\delta$). In this case, the isotherms become more thicker and take the shape as a horizontal lines along the interior cylinder. When the heat generation increases, the confusion in the isotherms becomes very strongly and it takes non-linear and irregular shape. This feature can be observed for all values of the location of the interior cylinder along the enclosure diagonal ($\delta$). As a result the amount of heat increases and it is transferred in this case by free convection emphasizing, that the buoyancy force effects which are responsible on the fluid movement become more dominant. Moreover, it can be noticed around the interior cylinder and adjacent to enclosure side walls increases also. Moreover, it can be shown that some isotherms cross through the interior cylinder. This means that an amount of heat which comes from the hot left sidewall can cross in its movement the interior cylinder and then returns to air inside the square enclosure. Furthermore, the flow vortices circulation increases as the heat generation increases. Figure 5 shows also, that the flow field behavior remains almost the same with the case of no heat generation for different interior cylinder along the enclosure diagonal locations ($\delta$). This means that the interior cylinder locations does not play an important role when the interior cylinder does not generate heat (i.e., $G = 0$). Figure 6, explains the variation of average Nusselt number of the enclosure at the hot left side wall with different locations of interior cylinder along the enclosure diagonal ($\delta$) for various Rayleigh numbers and heat generation. For the case with no heat generation ($G=0$), the figure shows that as the Rayleigh number increases, the average Nusselt number increases also. This due to the fact, that as Rayleigh number increases, the buoyancy effect increases and more confusion and disturbance occur in the isotherms. Therefore, as a result more amount of heat can be transferred in the enclosure leading to a clear increase in the average Nusselt number.

![Figure 4: Isotherm (left) and streamlines (right) for different values of G and \(\delta\) while K=5 and Ra=10^3.](image)
transfers from the hot left side wall to the right cold side wall leading to increase in the average Nusselt number at the cold wall and a decrease of it in the hot left side wall.

V. CONCLUSIONS

The following conclusions can be drawn from the results of the present work.

1. The results lead to conclude that a small effect of (δ) can be shown on the flow and thermal fields for the small range of Rayleigh number and (δ) and for a case of no heat generation.

2. The flow field behaviour changes clearly as a result of interior circular conductive cylinder movement, while the isotherms become almost the same, when the Rayleigh number is small (i.e, Ra = 10^3) and with no heat generation (G = 0).

3. The thickness of the hot fluid region around the interior cylinder increases as the heat generation values increase.

4. The thermal boundary layer can be observed around the interior cylinder and near the enclosure left and right side walls, and its thickness increases as the heat generation increases.

5. The heat generated by the interior cylinder influences significantly both the shapes of the stream contours and isotherm distributions. Also, it affects clearly on the thermal boundary layer thickness. This feature can be observed for all values of the location of the interior cylinder along the enclosure diagonal (δ).

6. When the Rayleigh number is low, the isotherm contours are in general take the shape of straight lines which are parallel to the enclosure side walls while the flow circulation is weak. As the Rayleigh number increases, the isotherms take the highly curved non-uniform lines shape. This can be observed for various values of heat generation and the interior cylinder along the enclosure diagonal (δ).

7. For the case with no heat generation (G=0), the results show that as the Rayleigh number increases, the average Nusselt number for both hot and cold side walls increase also with selected range of (δ).

8. The average Nusselt number at the hot left side wall decreases for all values of (δ) when the heat generation value increases. From the other hand, the average Nusselt number at the cold right side wall increases for all values of (δ) when the heat generation value and Rayleigh number increases.

Table (1) Comparison of present surface-averaged Nusselt number with those of previous studies.

<table>
<thead>
<tr>
<th>Ra</th>
<th>Mean Nusselt number at the hot wall</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^4</td>
<td>3.40470</td>
<td>3.4140</td>
</tr>
<tr>
<td>10^5</td>
<td>5.12893</td>
<td>5.1385</td>
</tr>
<tr>
<td>10^6</td>
<td>9.38875</td>
<td>9.3900</td>
</tr>
<tr>
<td>10^7</td>
<td>15.6995</td>
<td>15.665</td>
</tr>
</tbody>
</table>

Finally, Fig.7 shows the variation of average Nusselt number of the enclosure at the cold right side wall with various locations of interior cylinder along the enclosure diagonal (δ) for different Rayleigh numbers and heat cylinder along the enclosure diagonal (δ). This is because the high circulation of free convection currents causes an important increase in the amount of heat which means that no important effect of (δ) when the interior cylinder does not generate heat (i.e, G = 0). When the levels of heat generation (G) are taken in account which are (10, 50, 100). It can be observed from Fig.6, that as the heat generation value increases, the interior cylinder surface temperature becomes less than the hot air temperature and as a result the heat transfer rate represented by the average Nusselt number decreases.
Laminar natural convection in cavities

Figure 6: Total surface-average Nusselt number of the enclosure hot left side wall along the $\delta$ for different Rayleigh numbers and Reynolds numbers

Figure 7: Total surface-average Nusselt number of the enclosure cold right side wall along the $\delta$ for different Rayleigh numbers and Reynolds numbers

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_s$</td>
<td>Circular solid cylinder area</td>
<td>$m^2$</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravitational acceleration</td>
<td>$m/s^2$</td>
</tr>
<tr>
<td>$G$</td>
<td>Heat generation or temperature ratio</td>
<td></td>
</tr>
<tr>
<td>$K$</td>
<td>Solid - fluid thermal conductivity ratio</td>
<td></td>
</tr>
<tr>
<td>$k$</td>
<td>Thermal conductivity of fluid</td>
<td>$W/m°C$</td>
</tr>
<tr>
<td>$L$</td>
<td>Thermal conductivity of solid circular cylinder</td>
<td>$W/m°C$</td>
</tr>
<tr>
<td>$L_c$</td>
<td>Length or height of the enclosure</td>
<td>$m$</td>
</tr>
<tr>
<td>$N_{ave}$</td>
<td>Average Nusselt number</td>
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</tr>
<tr>
<td>$P$</td>
<td>Dimensionless pressure</td>
<td>$N/m^2$</td>
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<tr>
<td>$p$</td>
<td>Pressure</td>
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<td>$\alpha$</td>
<td>Thermal diffusivity</td>
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<tr>
<td>$\beta$</td>
<td>Volumetric coefficient of thermal expansion</td>
<td>$k^{-1}$</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>Dimensionless temperature</td>
<td></td>
</tr>
<tr>
<td>$\Theta_s$</td>
<td>Dimensionless temperature of interior cylinder surface</td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>The location of the interior cylinder along the diagonal of the square enclosure</td>
<td>$m$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Kinematic viscosity of the fluid</td>
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<tr>
<td>$\rho$</td>
<td>Density of the fluid</td>
<td>$kg/m^3$</td>
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REFERENCES


