Thermal Dispersion in Porous Media – A Review on Approaches in Experimental Studies

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Abstract—Thermal dispersion is an important topic in heat transfer applications of porous media. In order to determine heat transfer in a packed bed, the effective thermal conductivity including both stagnant conductivity and thermal dispersion conductivity coefficients should be known. A literature survey reveals that theoretical, numerical and experimental studies have been performed on determination of the effective thermal conductivity for the packed bed. In this study, firstly brief information on thermal dispersion is presented and then a review on the approaches of the reported experimental studies was performed. The employed experimental methods for determination of axial and transverse thermal dispersions were classified into three groups. Each experimental approach for determination of thermal dispersion coefficient is explained with presenting a reported experimental study in the literature.

Keywords—Heat transfer, Thermal dispersion, Porous media

I. INTRODUCTION

A porous medium is a composite medium containing interconnected voids or solid particles embedded into fluid containing medium. The applications of heat and fluid flow in porous media are faced in many industrial processes such as packed beds, heat exchangers and nuclear reactors etc. Filters and membranes are also typical examples of porous media. Water and oil flowing through rocks and soil are modeled by using hydrodynamic equations of porous media.

In a porous medium, fluid flows in the pores (or voids) between particles. Flow through the pores is a complex and 3-dimensional; hence, determination of velocity and temperature fields is very difficult and numerical modeling is very costly. That is why; some approaches are required to handle this adversity of heat and fluid flow in porous media. One of the widely used approaches is macroscopic approach. According to this approach, heat and fluid flow equations are established for a continuum domain involving the whole volume of porous media although a discontinuity exists in the porous media due to two different phases (i.e. solid and fluid);. This requires defining a volume averaged velocity, \( \langle \mathbf{u} \rangle \), and volume averaged temperatures for solid and fluid phases, \( \langle T \rangle^s \) and \( \langle T \rangle^f \).

Taking volume integral of continuity, momentum and energy equations yields governing equations for the whole of porous media. However, it causes appearing of extra terms in the governing equations. The volume average of energy equations causes addition of two extra terms as thermal dispersion and tortuosity. In this study, after presenting brief information on definition of thermal dispersion, experimental approaches reported in the literature for determination of thermal dispersion are reviewed.

II. THERMAL DISPERSION

From the micro point of view, the fluid velocity in the voids between the particles of porous media is not uniform and a discontinuity exists in the velocity and temperature fields because of solid particles. Although microscopic analysis of heat and mass flow has been reported in the literature, most of the researchers prefer to employ macroscopic approach.

For a porous medium with non thermal equilibrium, microscopically, the following energy equations are valid for the fluid and solid phases, respectively.

\[
\rho_f c_p^f \left( \frac{\partial T_f}{\partial t} + \boldsymbol{\nabla} \cdot \overline{\mathbf{u}} T_f \right) = k_f \nabla^2 T_f
\]

\[
\rho_s c_p^s \frac{\partial T_s}{\partial t} = k_s \nabla^2 T_s
\]

where \( \rho, c_p, T \) are the density, specific heat and temperature. The velocity and thermal conductivity coefficient are shown by \( \overline{\mathbf{u}} \) and \( k \). Subscripts \( f \) and \( s \) denote the fluid and solid phases, respectively.

To reach macroscopic relations, volume averaging approach is used. The pore and particle volume averages can be used separately to obtain the intrinsic averaged properties of fluid and solid phases. A detailed formulation of volume averaging approach is given in the appendix. As can be seen from this, macroscopic forms of energy equations for the solid and fluid phases can be easily obtained by using volume average method.
Thermal equilibrium assumption between solid and fluid phases is valid and widely used in thermal analysis of porous media. The summation of macroscopic energy equations of separate phases yields Eq. (3) which is valid for a porous continuum domain.

\[
\frac{\partial (\rho c_p)_{sf}}{\partial t} + \rho f c_{pf} \langle \vec{u} \rangle \cdot \nabla \langle T \rangle = k_{sf} \nabla^2 \langle T \rangle
\]

\[
- \nabla \cdot \left[ \frac{1}{V_s} \left( k_f - k_s \right) T dS \right] - \rho_f c_{pf} \nabla \cdot \langle T' \vec{u}' \rangle
\]

(3)

Here, \((\rho c_p)_{sf}\) and \(k_{sf}\) are equivalent thermal capacitance and thermal conductivity of the porous continuum domain including porosity as well as both solid and fluid thermal properties. The last two terms in the macroscopic energy equation have distinctive names as tortuosity and thermal dispersion. These terms do not exist in microscopic governing equations and they are resulted from volume integration.

The average of temperature and velocity fluctuations can be defined based on the temperature gradient in the porous continuum domain. Hence, the thermal dispersion term can be written in the form of a diffusion transport as given by Eq. (4) where \(k_{dis}\) is defined as thermal dispersion conductivity coefficient.

\[
k_{dis} \nabla^2 \langle T \rangle = -\rho_f c_{pf} \nabla \cdot \langle T' \vec{u}' \rangle
\]

(4)

The tortuosity term regards the change of thermal diffusion due to the micro-structure of the solid matrix. Under the above arrangement and neglecting the tortuosity term, the energy equation takes new form as presented by Eq. (5).

\[
\frac{\partial (\rho c_p)_{sf}}{\partial t} + \rho_f c_{pf} \langle \vec{u} \rangle \cdot \nabla \langle T \rangle = k_{eff} \nabla^2 \langle T \rangle
\]

(5)

where \(k_{eff}\) is the effective thermal conductivity which is the summation of equivalent thermal conductivity of the porous continuum domain and the thermal dispersion conductivity.

Thermal dispersion occurs in different directions of a porous medium due to non uniform effects of velocity and volume averaging of the temperature field. Since heat diffusion occurs in all directions, the effective thermal conductivity is a tensor whose diagonal terms represent longitudinal and transverse effective thermal conductivity of the porous continuum domain. This effective thermal conductivity depends on various parameters such as mass flow rate, porosity, shape of pores, generated temperature gradient, and solid and fluid thermal properties [2].

III. DETERMINATION OF THERMAL DISPERSION

When the macroscopic approach is employed to analyze heat transfer in a porous medium, the determination of an accurate value for effective thermal conductivity (or thermal dispersion conductivity) in the porous continuum domain is very important. Considerable numerical and experimental studies on the effective thermal conductivity in axial and transverse directions in various packed beds are reported in the literature. In the most of these researches, the effective thermal conductivity ratio which is the ratio of the effective thermal conductivity of the porous continuum domain to the fluid thermal conductivity has been determined. Most of the obtained relations depend on two parameters as particle Reynolds and Peclet numbers. Note that, for a porous media, the Reynolds number (or particle Reynolds number) is defined as \(Re = \frac{\rho f \langle \vec{u} \rangle d_p}{\mu_f}\) where \(\langle \vec{u} \rangle\) is superficial (volume averaged, Darcian) velocity, \(d_p\) is the equivalent particle diameter and \(\mu_f\) is dynamic viscosity of the fluid. The Peclet number is obtained by multiplying the defined Reynolds number with Prandtl number of the fluid. Numerical studies on the determination of the effective thermal conductivity ratio have been reported in literature, but the present study focus on the review of experimental studies in this area.

IV. EXPERIMENTAL STUDIES ON THERMAL DISPERSION IN POROUS MEDIA

Experimental methods have been developed to determine the effective thermal conductivity in porous media. Different porous materials and shapes have been used in the performed studies. Many researchers preferred to use spherical particles while some others used cylinders, Raschig rings, etc. The employed porous materials show variety as well. Most of researchers have used spherical glass particles, while ceramic and steel materials have also been used. As for the working fluid, air is used in majority of the experiments nevertheless water also has been used in some of the investigations.

Experimental procedures used in many experiments are basically very similar. Simply experimental studies on the determination of the effective thermal conductivity are generally performed by temperature measurement at various locations of a packed bed (or porous media) when a heat input is imposed. Some investigators preferred to determine the effective thermal conductivity under steady state circumstances while others used transient state. A heat source used to generate temperature gradient in the bed has been applied inside the packed bed (for example at the centerline) or imposed at the bed boundaries. To determine the effective thermal conductivity which includes both equivalent thermal conductivity of porous media and thermal dispersion conductivity, usually the following procedure has been followed:

- A temperature gradient in the packed bed has been generated by using a heat source/sink at the bed boundaries or inside the bed. Temperatures at the different packed bed locations have been measured.
- The macroscopic energy equation (i.e., Eq. (5)) is solved for the packed bed. In the most of the studies, analytical methods have been used to obtain solution of Eq. (5).

- Finally, the effective thermal conductivity for the considered packed bed has been obtained by comparison of the obtained analytical and experimental results of the temperature field.

The literature survey shows that the experimental studies for determination of axial and transverse thermal dispersion can be classified into three groups as a) Heat Addition/Removal at Lateral Boundaries, b) Uniform Inlet Temperature, c) Heat Addition inside the Bed. In the following sections, these approaches have been explained with presenting an experimental study reported in literature.

A. Heat Addition/Removal at the Lateral Boundaries

This method is widely used in recent researches. In this approach, temperature gradient in the bed is generated by imposing uniform temperature or heat flux at the lateral boundaries of the packed bed. By the other words, temperature gradient in the radial and transverse directions in the bed has been generated by adding or removing heat from the lateral surface of the packed bed. Literature survey shows that the cylindrical packed bed has been mostly used to provide axisymmetrical boundary condition. The use of axisymmetrical packed bed provides advantages such as checking of measured temperature at different locations and simplifying heat transfer equation (i.e., Eq. 5). Steady state results have been used in the most researches of Heat Addition/Removal at the Lateral Boundaries approach. Comparison of solution of the steady state macroscopic energy equation (i.e., Eq. 5) with the experimental results at steady state yields effective thermal conductivity values.

The study performed by Smirnov et al. [3] can be given as an example for this kind of experimental researches. They studied steel and glass spheres, ceramic cylinders and ceramic and copper Raschig rings as packings for the cylindrical packed bed shown in Figure 1. They investigated Reynolds numbers in the range of 250 to 2250. They neglected the diffusion in the axial direction. As seen from Figure 1, fluid enters to the packed bed with a specified mass flow rate firstly form the heating section to raise its temperature to a specified value. Then, it passes through the calming section providing fully developed velocity field. After a calming section, it enters to the main section whose lateral surface temperature is different than fluid temperature. The heat removal/addition has been done by circulating water in an annular jacket around the test section.

The related energy equation for Heat Addition/Removal at the Lateral Boundaries approach has been given by Eq. (6) where \( k_r \) and \( k_{eax} \) are effective radial and axial thermal conductivities, respectively. Boundary conditions are: (1) constant temperature at the inlet; (2) symmetry at the center of the packed bed; (3) constant temperature that equals to lateral wall temperature at the outlet; and (4) convective heat transfer at the lateral surface.

\[
\left( \rho c_p \right)_f \langle u \rangle \frac{\partial \langle T \rangle}{\partial z} = k_e \left[ \frac{\partial^2 \langle T \rangle}{\partial r^2} + \frac{1}{r} \frac{\partial \langle T \rangle}{\partial r} \right] + k_{eax} \frac{\partial^2 \langle T \rangle}{\partial z^2} \quad (6)
\]

In their study, the effective thermal conductivity and related wall heat transfer coefficient were found. They indicated that the radial effective thermal conductivity can be predicted with Eq. (7) where the convective heat transfer parameter, \( K \), depends on the various properties of the porous structure and flow. They obtained some experimental values for \( K \) and wall heat transfer coefficient. The following equation has been proposed to determine effective thermal conductivity.

\[
\frac{k_{er}}{k_f} = \frac{k}{k_f} + K \operatorname{Re} \operatorname{Pr} \quad (7)
\]

They found \( k \) values as 0.089 for steel spheres (porosity of 0.41); 0.091 for glass spheres (porosity of 0.42); 0.146 and 0.14 for ceramic cylinders of porosity 0.40 and 0.42, respectively; 0.16 and 0.21 for ceramic Raschig rings of porosity 0.16 and 0.21, respectively.

B. Uniform Inlet Temperature Approach:

This approach was mostly used in the early studies of heat transfer in porous media. In this method, radially uniform temperature change can be obtained by placing a grid heater at the inlet section of the packed bed. Heater can be arranged in a way that its heat output changes with time. Hence, a pulse heat input or a sinusoidal temperature variation in time can be
achieved. For a step change, a sudden temperature change is applied at the inlet boundary and then the temperature remains constant. For a pulse change, a finite amplitude pulse is applied by the heater and it is repeated periodically. With a similar arrangement sinusoidal change in bed temperature can be achieved. The measurements are made for downstream temperature change and phase difference between the waves at the inlet and outlet of the packed bed. Hence the response of the bed to inlet temperature disturbances is obtained. The radial walls of the bed are insulated to decrease radial heat flux, thus the axial heat flux becomes dominant. Because of these assumptions one-dimensional transient form of macroscopic energy equation (Eq. 5) is used to obtain analytical solution of the temperature. At the inlet boundary, time varying form of the temperature is used, e.g. sinusoidal wave with constant amplitude. Initially constant temperature is assumed within the bed. Constant temperature boundary is also used at the outlet.

\[
\left( \rho c_p \right) \langle u \rangle \frac{\partial \langle T \rangle}{\partial z} = k_{e ax} \frac{\partial^2 \langle T \rangle}{\partial z^2}
\]  

Figure 2 – Experimental setup of Lindauer et al (Uniform Inlet Temperature Approach) [4].

Lindauer [4] studied heat transfer in packed beds by using sinusoidal temperature variations. Air is used as a working fluid in a rectangular packed bed filled with steel and tungsten spheres. The experimental setup is shown in Figure 2 in which, a calming section was used before applying heat to the fluid. A cyclic power generator and a resistance heater are also used prior to test section to create sinusoidal temperature waves. Then amplitudes and phase changes of temperature wave are measured in order to determine the particle-to-coolant heat transfer coefficient. Studied Reynolds numbers of this research were in between 23.4 and 18200. The effective thermal conductivity of the packed bed was not found explicitly, instead, Colburn j factor of heat transfer for varying mass flow rates and Reynolds numbers were calculated.

C. Heat Addition inside the Bed:

In this method, a plane or a point heat source is placed inside the packed bed. Time dependent or steady heat sources can be used within usually rectangular packed beds. Thus a wire that is placed perpendicular to the fluid flow can be thought as a point source and several wires in the same plane can be thought as a plane heat source inside the packed bed. Both axial and transverse components of the effective thermal conductivity can be determined if thermocouples distributed throughout the packed bed.

The governing equation the aforementioned experimental setup is given in Eq. (9) where \( k_{etr} \) is effective transverse thermal conductivity and \( s \) is source strength. The boundary conditions for this equation are constant temperature at the inlet, thermally developed outlet in which temperature gradient is kept zero and the insulated lateral boundaries. The initial condition is a constant reference temperature.

\[
\left( \rho c_p \right)_f \frac{\partial \langle T \rangle}{\partial t} + \rho_f c_p \langle u_f \rangle \frac{\partial \langle T \rangle}{\partial x} = k_{e ax} \frac{\partial^2 \langle T \rangle}{\partial x^2} + k_{e tr} \frac{\partial^2 \langle T \rangle}{\partial y^2} + s
\]  

Figure 3 : Experimental setup of Metzger et al. a) point heat source, b) plane heat source [5].

Metzger et al. [5] investigated a rectangular packed bed with glass spheres whose experimental setups are shown in Figure
Water was flowing from top to bottom in the packed bed with constant inlet temperature and Peclet numbers were kept below 130. A single wire near the inlet of the packed bed is used as a point heat source then more heaters in a same plane is placed to create a plane heat source. Researchers claimed that the method used is good to predict axial effective thermal conductivity, but the lateral component cannot be determined well. They proposed Eq. (10) for the axial effective thermal conductivity.

\[
\frac{k_{\text{ax}}}{k_f} = \frac{k}{k_f} + 0.073Pe^{1.59}
\]  

(10)

V. Conclusion

The experimental studies have been performed for determination of transverse and axial thermal dispersion conductivities can be classified into three groups as a) Heat Addition/Removal at Lateral Boundaries, b) Uniform Temperature at Inlet Boundary, and c) Heat Addition Inside Bed. The approach of Heat Addition/Removal at Lateral has been performed under steady state conditions on the other hand, the Uniform Temperature at Inlet Boundary approach has been used at transient state. The approach of Heat Addition inside the bed can be achieved by transient or steady states.

Generally, the procedure is almost the same for all these experimental approaches. A temperature gradient is generated in the bed. The temperature is measured at various locations in the packed bed. An analytical solution for the macroscopic energy equation is obtained under the imposed boundary/initial conditions. Then, the comparison between the experimental and analytical values of temperature distributions in the bed yields the effective thermal conductivity. The equivalent thermal conductivity of the bed can be calculated by using thermal conductivity of fluid and solid and porosity. Thus, if one requires thermal dispersion conductivity, it can be found by subtracting of effective and equivalent thermal conductivities.

Appendix

Volume averaging of a quantity \( \phi \) is calculated from Eq. (A1) where \( V \) is total volume of fluid and solid phases.

\[
\langle \phi \rangle = \frac{1}{V} \int \phi dV
\]  

(A1)

To separately obtain the intrinsic averaged properties of two phases, the pore volume averaging is considered for fluid phase (Eq. A2) and the particle volume averaging is considered for solid phase (Eq. A3) [1].

\[
\langle \phi \rangle^f = \frac{1}{V_f} \int \phi dV
\]  

(A2)

\[
\langle \phi \rangle^s = \frac{1}{V_s} \int \phi dV
\]  

(A3)

The volume averaged velocity is called Darcian or superficial velocity which is found by using Eq. (A4).

\[
\langle u \rangle = \frac{1}{V} \int udV = \frac{1}{V} \int udV
\]  

(A4)

The fluctuation of a quantity is defined as

\[
\phi' = \phi - \langle \phi \rangle
\]  

(A5)

Also, the below property is used for the volume average of the multiplication of two properties.

\[
\langle \phi_1 \phi_2 \rangle = \langle \phi_1 \rangle \langle \phi_2 \rangle + \langle \phi'_1 \phi'_2 \rangle
\]  

(A6)

By using the definition of volume averaging and corresponding rules, the macroscopic energy equations can be found as Eq. (A7) for fluid phase and Eq. (A8) for solid phase where \( \varepsilon \) is porosity and defined as the ratio of fluid volume to total volume.

\[
\rho_f c_{p_f} \left( \varepsilon \frac{\partial \langle T \rangle^f}{\partial t} + \langle u \rangle \cdot \nabla \langle T \rangle^f \right) = k_f \varepsilon \nabla^2 \langle T \rangle^f + \nabla \cdot \left[ \frac{1}{V_s} \int k_f TdS \right]
\]  

(A7)

\[
\rho_s c_{p_s} (1-\varepsilon) \frac{\partial \langle T \rangle^s}{\partial t} = k_s (1-\varepsilon) \nabla^2 \langle T \rangle^s - \nabla \cdot \left[ \frac{1}{V_s} \int k_s TdS \right] - \frac{1}{V_s} \int k_f \nabla TdS
\]  

(A8)

If thermal equilibrium between two phases is assumed the summation of (A7) and (A8) gives Eq. (3) for a porous continuum domain. The effective thermal capacitance and equivalent thermal conductivity are defined as:

\[
\rho_{\text{eff}} c_{\text{p, eff}} = (1-\varepsilon) \rho_f c_{p_f} + \varepsilon \rho_s c_{p_s}
\]  

(A9)

\[
k_{\text{eff}} = (1-\varepsilon) k_s + \varepsilon k_f
\]  

(A10)
REFERENCES


